

PART 1. Part 1 consists of 6 questions. Do your work and clearly write your answer in the space provided. No partial credit is awarded for this part of the test. (5 points each)

1. Write out the form of the partial fraction decomposition of the function $f(x) = \frac{4x}{(x-1)^2(x^2+1)}$.

Do not determine the numerical values of the coefficients.

Answer: _____

2. Find the derivative of the function $g(x) = \int_1^x \sqrt{8+t^3} dt$

Answer: _____

3. Evaluate the integral $\int_1^e \frac{\ln x}{x} dx$.

Answer: _____

4. Evaluate the definite integral $\int_0^3 \sqrt{9-x^2} dx$ by interpreting it in terms of area.

Answer: _____

5. Evaluate the indefinite integral $\int \tan x \sec^4 x dx$

Answer: _____

6. Evaluate the indefinite integral $\int x \ln x dx$.

Answer: _____

Part 2. Part 2 consists of 5 problems worth 14 points apiece. Show all your work for full credit! Displaying only the final answer (even if correct) without the relevant steps is not enough.

Problem 1

- a. Write out the Riemann sum for $f(x) = \frac{1}{x}$, $1 \leq x \leq 5$, with a regular partition and four terms, taking the sample points to be midpoints. (You do not actually have to add up the terms of the sum). What does the Riemann sum here represent? Illustrate with a diagram.

- b. Evaluate $\int_1^5 \frac{1}{x} dx$. Geometrically, what does the integral represent? Illustrate with a diagram.

Problem 2

A particle moves along a line with velocity function $v(t) = t^2 - t$ where v is measured in meters per second.

(a) Find the change in position, i.e. the displacement, over the time interval $[0,3]$.

(b) Find the distance traveled by the particle during the time interval $[0,3]$

Problem 3

(a) Using the substitution $x = \sin \theta$, where $-\pi/2 < \theta < \pi/2$, evaluate the integral

$$\int \frac{x^3}{\sqrt{1-x^2}} dx.$$

(b) Evaluate the indefinite integral $\int \frac{x+1}{x^2+1} dx$ (Hint: Break up the fraction in the integrand into a sum of two fractions.)

Problem 4

(a) Evaluate the indefinite integral $\int x^2 e^x dx$.

(b) Find the average value of $f(t) = te^{-t^2}$ over the interval $[0,2]$.

Problem 5

(a) Evaluate the definite integral. $\int_1^4 \frac{x^3 + \sqrt{x}}{x^2} dx$.

(b) Evaluate the indefinite integral $\int \frac{1}{x^2 - x - 2} dx$