

MA 125 - 6C, CALCULUS I

October 05, 2009

Name (Print):

Student Signature:

TEST II

No calculators are allowed!

PART I

Part I consists of 10 questions. Clearly write your answer in the space provided after each question. Show your work as much as possible.

Each question is worth 5 points.

Question 1

Differentiate the function $y = (2 + x)(2 - x)$. (Simplify your answer!)

Answer: $y' = -2x$.

Question 2

Differentiate the function $y = (5 + \sqrt{x})^{25}$.

Answer: $y' = \frac{25(5+\sqrt{x})^{24}}{2\sqrt{x}}$

Question 3

Find $f'(x)$ if $f(x) = \frac{x+2}{x-1}$.

Answer: $y' = -\frac{3}{(x-1)^2}$

Question 4

Differentiate the function $f(t) = t^2 \sin(t)$.

Answer: $f' = 2t \sin(t) + t^2 \cos(t)$

Question 5

Find $y'' = \frac{d^2y}{dx^2}$ if $y = x^3 + \cos(x)$.

Answer: $y'' = 6x - \cos(x)$

Question 6

Differentiate the function $y = \sqrt{2x+3}$.

Answer: $y' = \frac{1}{\sqrt{2x+3}}$

Question 7

Differentiate the function $g(\theta) = \tan(6\theta)$.

Answer: $g' = 6 \sec^2(6\theta)$

Question 8

Differentiate the function $f(x) = \sqrt{\sin(x)}$.

Answer: $f' = \frac{\cos(x)}{2\sqrt{\sin(x)}}$

Question 9

Differentiate the function $y = \sin(x^3)$.

Answer: $y' = 3x^2 \cos(x^3)$

Question 10

If $S(t) = t - t^2$ is the position of a particle at time t , find the velocity at time $t = 1$.

Answer: -1

PART II

Each problem is worth 10 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

Find the equation of the tangent line to the graph of the function $y = f(x) = x^2(x - 2)^3$ at the point with $x = 1$.

Answer: $y = x - 2$

Problem 2

Find y' if $x^3 + y^2 = 2x^2y$.

Answer:

$$y' = \frac{3x^2 - 4xy}{2x^2 - 2y}$$

Problem 3

Find the derivative of the function $f(x) = \frac{x^2 + 1}{x^2 - 1}$. You must simplify the fraction.

Answer:

$$y' = -\frac{4x}{(x^2 - 1)^2}$$

Problem 4

Find all the x -values where the graph of the function $y = f(x) = \frac{x^4}{4} - \frac{x^2}{2}$ has a horizontal tangent line.

Answer: $x = 0$, $x = 1$, and $x = -1$, which are solutions to the equation $y' = x^3 - x = 0$

Problem 5

Air is leaking from a spherical balloon so that its volume decreases at a rate of $2 \text{ cm}^3/\text{s}$. How fast is the radius of the balloon decreasing when its diameter is 20 cm?

Answer:

$$\frac{dR}{dt} = -\frac{1}{200\pi}$$