FALL 2010 — MA 227 — FINAL EXAM
FRIDAY DECEMBER 10, 2010

NAME: ____

THERE ARE 11 QUESTIONS, EACH WORTH 10 POINTS; 100 (OR MORE) POINTS IS EQUIVALENT TO 100% FOR THE EXAM. PARTIAL CREDIT IS AWARDED WHERE APPROPRIATE. SHOW ALL WORKING; YOUR SOLUTION MUST INCLUDE ENOUGH DETAIL TO JUSTIFY ANY CONCLUSIONS YOU REACH IN ANSWERING THE QUESTION.

1. (a) Find the equation of the plane containing the points $(1,1,2)$, $(0,1,-1)$ and $(-1,2,1)$.
   (b) Let $\mathbf{r}(t) = (2t^2, \sin(t^3-1), 2)$. Find the unit tangent vector at the point on the curve corresponding to $t=1$.

2. (a) Let $f(x, y, z) = xz\cos(y) - xyz^2$. Find the third partial derivative f'''_{xyz}.
(b) Let $f = xy^2z$ and $\mathbf{F} = (xz,\, y,\, x^2z)$. Find ∇f (the gradient of f), $\operatorname{div}\mathbf{F}$ (the divergence of $\mathbf{F}$), and $\operatorname{curl}\mathbf{F}$ (the curl of $\mathbf{F}$).

3. (a) Find the directional derivative of the function $f(x, y, z) = yz - xy$ in the direction of the vector $\vec{v} = 2\vec{i} - \vec{j} + 2\vec{k} = \langle 2, -1, 2 \rangle$ at the point $(1, 1, -1)$.
   (b) Find the maximum rate of change of $f(x, y) = x^2 y + 2\sqrt{x}$ at the point $(1, 1)$. In which direction does it occur?

4. (a) Let $z = x^2y - x^3$. Find the equation of the tangent plane at the point $(1, 2)$.
   (b) Find equation of the tangent plane to the surface $x + 2y^2 - z^3 = 3$ at the point $(2, -1, 1)$.

5. Find the local maximum, minimum and saddle points (if any) of the function

$$f(x, y) = x^2 - xy + y^2 + 9x - 6y + 10.$$

6. (a) Find the linear approximation for the function

$$f(x,y) = ye^{x-y} - x^2y^2$$

near the point $(1,1)$.

(b) Let $f(x,y) = xy^3 - ye^x$ and $x = s + t^2, \quad y = st$. Find the partial derivatives $\partial f/\partial s$ and $\partial f/\partial t$. You don't need to simplify your answer!

7. Find the absolute maximum and absolute minimum points of the function

$$f(x, y) = x^2 + y^2 + x$$

on the region $-1 \leq x \leq 1, \quad -1 \leq y \leq 1$. Be sure to provide the coordinates of the points and the values of absolute maximum and minimum.

8. Evaluate, by making an appropriate change of variables, the integral

$$\iint_D (x+y)^2 e^{x-2y}\, dA$$

where D is the parallelogram enclosed by the lines $x-2y=0$, $x-2y=2$, $x+y=-1$, and $x+y=1$.

9. (a) Switch the order of integration in the iterated integral

$$\int_0^1 \left[\int_0^{2x} f(x,y)\,dy \right] dx.$$

(b) Using a double integral, find the area of the triangle with vertices $(0,0)$, $(2,1)$, $(1,2)$.

10. (a) Change $(1, \sqrt{3}, 2\sqrt{3})$ from rectangular into spherical coordinates.
(b) Using spherical coordinates evaluate

$$\int\int\int_E (x^2 + y^2 + z^2)\, dV,$$

where E is the half-ball $x^2 + y^2 + z^2 \leq 4$, $z \geq 0$.

11. Use polar coordinates to find the mass of the lamina that lies within the annual region $1 \leq x^2 + y^2 \leq 16$, if the material in the lamina has density (mass per unit volume) given by $\rho(x, y) = x^2 + y^2$.