

MA 126-8C, CALCULUS II

Thursday, November 29, 2012

Name (Print last name first):

Student Signature:

TEST IV

Closed book - No calculators!

PART I

Each question is worth 5 points.

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You need not show your work for this part of the test. No partial credit is awarded for this part of the test!

Question 1

Determine whether the **sequence** $a_n = (-1)^n \frac{n+8}{n^2+6}$ converges or diverges. If it converges, find its limit.

Answer:

Question 2

Find the limit of the **sequence** given by $a_n = \ln(e^{1/n})$. (Your answer must be a number!)

Answer:

Question 3

Determine whether the **geometric series** $\sum_{n=0}^{\infty} \left(\frac{5}{4}\right)^n$ is convergent or divergent. If it is convergent, find its sum.

Answer:

Question 4

Determine whether the infinite **series** $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{n+2}$ is convergent or divergent.

Answer:

Question 5

Use the integral test to determine whether the infinite **series** $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ is convergent or divergent.

Answer:

Question 6

Determine whether the **alternating series** $\sum_{n=1}^{\infty} (-1)^n \frac{n^2+1}{n^4+6}$ is divergent, absolutely convergent, or conditionally convergent.

Answer:

PART II

Each problem is worth 14 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit - no credit for unsubstantiated answers!

Problem 1

Determine whether or not each of the following **sequences** converges as $n \rightarrow \infty$. (Give reasons!)

$$a_n = \frac{e^{-n}}{n+1}$$

$$b_n = \sin\left((2n-1)\frac{\pi}{2}\right)$$

$$c_n = \frac{n}{\ln(n)}$$

Problem 2

This problem has two separate questions. Answer each question!

- (a) Find the numerical value of c for which (the infinite series)

$$\sum_{n=1}^{\infty} \frac{1}{(1-c)^{n-1}} = 3/2.$$

- (b) Find the values of x for which the **geometric series**

$$\sum_{n=0}^{\infty} 4^n (x+2)^n$$

converges? Write your answer in interval notation!

Problem 3

Find the radius and interval of convergence of the **power series**

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}} (x+3)^n.$$

Be sure to check any endpoints that exist!

Problem 4

Answer all the following questions.

- (a) Find a power series representation of the function

$$f(x) = \frac{1}{1+x^2},$$

and state the interval on which the series converges to the function.

- (b) Use the series in (a) to find a power series representation of the function

$$g(x) = \frac{x^5}{1+x^2}.$$

Problem 5

Answer all the following questions.

- (a) Find the Maclaurin series representation of the function $f(x) = \sin(x^2)$. (Hint: The Maclaurin series of $\sin(x)$ might prove useful here, if need be!)

- (b) Use the series in (a) to evaluate the integral

$$\int \sin(x^2) dx$$

as a power series.

- (c) Use the series in (b) to write out the Maclaurin **series representation** of

$$\int_0^1 \sin(x^2) dx$$

(Do not compute and add the terms of your series!)

- (d) Find the minimum number of terms you need in the series in (c) to approximate $\int_0^1 \sin(x^2) dx$ with an error less than $10^{-3} = 0.001$? (Show your work!)

SCRATCH PAPER

(Scratch paper will not be graded!)

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