FALL 2012 — MA 227 — FINAL EXAM
SATURDAY, DECEMBER 08, 2012

NAME: ____

THERE ARE 10 QUESTIONS, EACH WORTH 11 POINTS; 100 (OR MORE) POINTS IS EQUIVALENT TO 100% FOR THE EXAM. PARTIAL CREDIT IS AWARDED WHERE APPROPRIATE. SHOW ALL WORKING; YOUR SOLUTION MUST INCLUDE ENOUGH DETAIL TO JUSTIFY ANY CONCLUSIONS YOU REACH IN ANSWERING THE QUESTION.

1. (a) Find the equation of the plane containing the points $(1, -1, 2)$, $(1, 1, -1)$ and $(-1, 0, 1)$.
   (b) Let $\mathbf{r}(t) = (t^3,\ 1,\ e^{t^2-1})$. Find the unit tangent vector at the point on the curve corresponding to $t = 1$.

2. (a) Find the directional derivative of the function $f(x, y, z) = xz - yz^2$ in the direction of the vector $\vec{v} = \vec{i} + \vec{j} - 2\vec{k} = \langle 1, 1, -2 \rangle$ at the point $(-1, 0, 1)$.
   (b) Find the maximum rate of change of $f(x, y) = xy^3 - 4\sqrt{x}$ at the point $(1, -1)$. In which direction does it occur?

3. (a) Let $z = y^2 - x^3y$. Find the equation of the tangent plane at the point $(1, -2)$.
   (b) Find equation of the tangent plane to the surface $x^2 + 2y - xz^2 = 4$ at the point $(1, 2, 1)$.

4. Find the local maximum, minimum and saddle points (if any) of the function

$$f(x, y) = x^2 - 4xy + y^2 - 2y + 2.$$

5. (a) Find the linear approximation for the function

$$f(x,y) = ye^{x-2} - x^2y^3$$

near the point $(2,1)$.

(b) Let $f(x,y) = x^2y - e^y$ and $x = s - t$, $y = s^2t$. Find the partial derivatives $\partial f/\partial s$ and $\partial f/\partial t$. You don't need to simplify your answer!

6. Find the absolute maximum and absolute minimum points of the function

$$f(x, y) = x^2 - y^2 + y$$

on the region $0 \leq x \leq 1, \quad -1 \leq y \leq 1$. Be sure to provide the coordinates of the points and the values of absolute maximum and minimum.

7. Evaluate the integral

$$\iint_D (x-y)^3 \sin(x+2y)\, dA$$

where D is the parallelogram enclosed by the lines $x-y=0$, $x-y=1$, $x+2y=0$, and $x+2y=\frac{\pi}{4}$. Use the change of the variables $u=x-y$, $v=x+2y$.

8. (a) Switch the order of integration in the iterated integral

$$\int_0^1 \left[\int_{x^2}^{x} f(x,y)\,dy \right] dx.$$

(b) Using a double integral, find the area of the triangle with vertices $(0,0)$, $(1,1)$, $(0,2)$.

9. (a) Change $(1, -\sqrt{3}, -2\sqrt{3})$ from rectangular into spherical coordinates.
(b) Using spherical coordinates evaluate

$$\int\int\int_E (x^2 + y^2 + z^2)^2 \, dV,$$

where E is the half-ball $x^2 + y^2 + z^2 \leq 1$, $x \leq 0$.

10. Use polar coordinates to find the mass of the lamina that lies within the region $x^2 + y^2 \leq 4$, $0 \leq x \leq y$, if the material in the lamina has density (mass per unit volume) given by $\rho(x, y) = x^2 + y^2$.