EGR 265-6D, Math Tools for Engineering Problem Solving

December 12, 2014, 1:30pm to 4:00pm

Name (Print last name first): . . . .

Student ID Number: . . . .

Final Exam

	Max. Score	Score
Problem 1	10	
Problem 2	10	
Problem 3	14	
Problem 4 (incl. Bonus)	10 + 5*	
Problem 5	14	
Problem 6	10	
Problem 7	12	
Problem 8	12	
Problem 9	8	
Total (incl. Bonus)	100 + 5*	

Problem 1 (10 points)

Find an explicit solution of the initial value problem

$$yy' = x\cos(x^2), \quad y(0) = -1.$$

Problem 2 (10 points)

Butter melts at 90°F. On a hot summer day you take a stick of butter out of the refrigerator, which is set to 50°F, and put it on your front porch where the temperature is 100°F. There it warms up according to Newton's law of cooling (which also applies to warming problems) and reaches a temperature of 70°F after 10 minutes. Note: In parts (b) and (c) logarithms do not need to be evaluated.

(a) Write down the IVP governing the warming process using an unknown rate k.

(b) Solve the IVP and determine k by using information provided in the problem.

(c) How long does it take before the butter melts?

Problem 3 (14 points)

Consider the second order differential equation

$$y'' + 4y' + 4y = \cos x + \sin x \qquad (1)$$

(a) Find the general solution of the homogeneous equation corresponding to (1).

(b) Find the general solution of (1).

Problem 4 (10 points + 5 points bonus)

A mass of 4 kg stretches an undamped spring by 40 cm. For simplicity, assume that $g = 10$ m/s^2.

(a) Find the spring constant k. Also find the angular frequency ω of the spring-mass system.

(b) Set up the second order differential equation which governs the motion of the spring-mass system, choosing the x-axis to be oriented downwards. Find the general solution of this equation.

(c) Find the particular solution of the equation if the mass is released from 50 cm above the equilibrium position at a downward velocity of 1 m/s.

(d) (Bonus) Suppose you take the mass and spring from above to the surface of Mars, where it holds apprriximately that $g = 4\ \mathrm{m/s}^2$. By how much does the mass stretch the spring there? How does the result of part (b), the general solution of Newton's equation, change?

Problem 5 (14 points)

(a) Find the gradient of $f(x, y) = \ln(x^2 + y)$.

(b) Find a unit vector in the direction of steepest descent of $f(x, y)$ at the point $(1, 1)$. Also find the rate of descent in this direction.

(c) Find parametric equations for the normal line to the graph of $z = f(x, y)$ through the point $(1, 1, \ln 2)$.

Problem 6 (10 points)

Find the line integral

$$\int_C xy^3\,ds,$$

where C is a quarter of a circle of radius 2, centered at the origin and contained in the first quadrant, starting at $(2,0)$ and ending at $(0,2)$.

Problem 7 (12 points)

(a) Verify that the force field $\mathbf{F}(x, y) = (ye^x - e^y + 1)\mathbf{i} + (e^x - xe^y)\mathbf{j}$ is conservative.

(b) Find a potential function $\phi(x, y)$ for $\mathbf{F}(x, y)$.

(c) Find the work done by the force field $\mathbf{F}(x, y)$ along the curve parameterized by $x = 1 - t^2$, $y = t^3$, $0 \leq t \leq 1$.

Problem 8 (12 points)

Let R be the triangular region in the xy-plane bordered by the three lines $y = x$, $y = -x$ and $y = 3$, and let $(\bar{x}, \bar{y})$ be the centroid of R.

(a) Sketch the region R and use a symmetry argument to determine one of the two coordinates of the centroid. Give a reason for your answer!

(b) Find the other coordinate of the centroid of R.

Problem 9 (8 points)

An inhomogeneous lamina of mass density $\rho(x, y) = x^2 + y^2$ fills the washer shaped region between the two disks of radius 1/2 and 1, both centered at the origin. Find the mass of the lamina.