MA 225 VT, HONORS CALCULUS I

October 14, 2015

Name (Print last name first): ....

Show all your work and justify your answer!

No partial credit will be given for the answer only!

PART I

You must simplify your answer when possible.
All problems in Part I are 10 points each.

1. Find the derivative of the function $y = f(x) = \cos(x^3)$.

2. Find the derivative of $f(x) = (x^2 + x)^8$.

3. Find the absolute maximum and minimum of the function $y = f(x) = (2x-3)^2(x+1)^5$ on the interval $[0, 1]$.

4. Find the linearization of the function $f(x) = x\tan(x)$ at the point $a = \pi/4$ and use it to estimate the value $f(.8)$.

5. Find two positive numbers so that their sum is 200 and their product is maximal. [As always you must justify your answer!]

6. Suppose that the **derivative** of a function $y = f(x)$ is given:

$f'(x) = (x + 2)(3 - x)$.

(a) Find the x-coordinates of all local max/min of the function $y = f(x)$.

(b) At which x value is the function $y = f(x)$ most rapidly increasing?

PART II

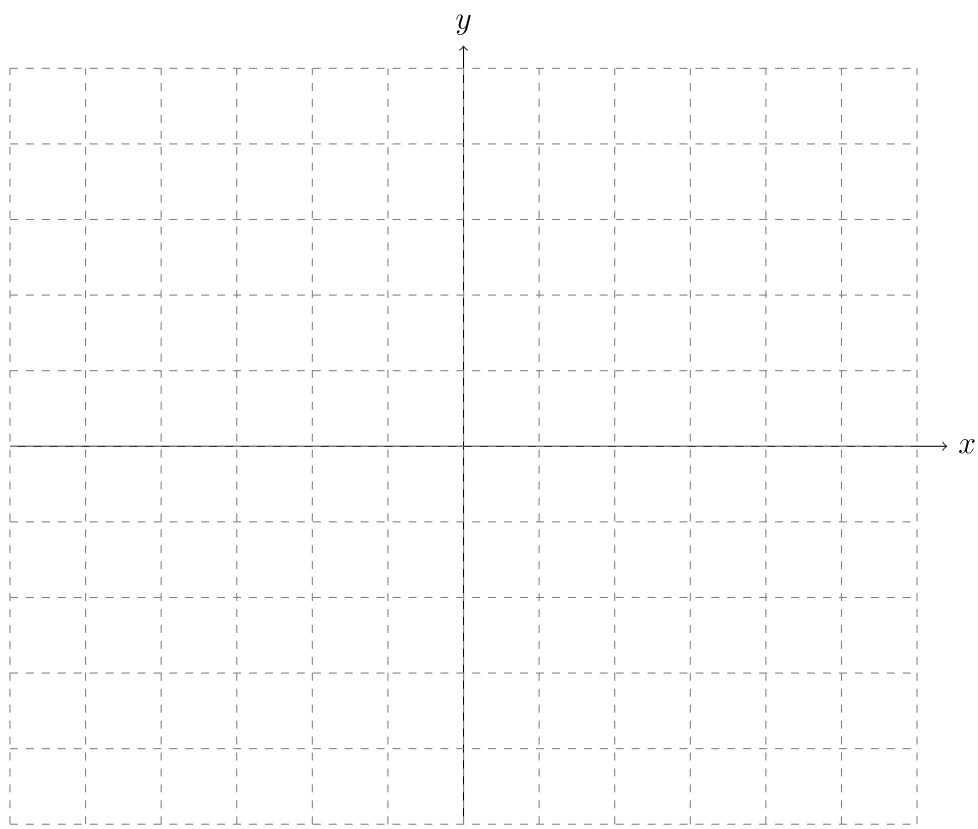
7. [**15 points**] You work for a soup company. In order to maximize visibility of the product on the shelve your boss asks you to design a soup can of volume $1\,dm^3$ and maximal surface area. Either specify the dimensions of such a can or show that such a can does not exist.

   You may use that the volume of a can of radius r and height h is $V = \pi r^2 h$ while the surface are of the side is $2\pi r h$ and of the top (and bottom) is πr^2.

8. [**20 points**] **Use calculus** to graph the function $y = f(x) = \dfrac{x}{x^2+1}$. Indicate

- x and y intercepts,
- vertical and horizontal asymptotes (if any),
- in/de-creasing; local/absolute max/min (if any).

You must show work to justify your graph and conclusions. You can use decimal numbers to plot points (but mark them with exact values).



9. [**5 points**] Find the equation of the tangent line to the graph of $x^2 + y^3 = 2xy$ at the point $(1, 1)$.