FALL 2015 — MA 227 — FINAL EXAM
WEDNESDAY, DECEMBER 09, 2015

NAME: ____

THERE ARE 15 QUESTIONS, EACH WORTH 7 POINTS; 100 (OR MORE) POINTS IS EQUIVALENT TO 100% FOR THE EXAM. PARTIAL CREDIT IS AWARDED WHERE APPROPRIATE. SHOW ALL WORKING; YOUR SOLUTION MUST INCLUDE ENOUGH DETAIL TO JUSTIFY ANY CONCLUSIONS YOU REACH IN ANSWERING THE QUESTION.

1. Let $f(x,y) = ye^x - \frac{\cos(y)}{x}$. Find the second partial derivative $f_{xy} = \frac{\partial^2 f}{\partial y \partial x}$.

2. Find local maximum, minimum and saddle points (if any) of the function
$$f(x,y) = x^2 + 4xy + y^2 - 2x + 1.$$

3. Let $z = x\cos(y) + y$. Find equation of the tangent plane at point $(1, 0)$.

4. Find the maximum rate of change of $f(x, y) = x^4 - \sqrt{\frac{x}{y}}$ at the point $(1, 1)$. In which direction does it occur?

5. Find the area of the region D bounded by $y = x^2$ and $y = 3x$.

6. Sketch the region of integration and change the order of integration:

$$\int_0^1 \int_{x^2}^2 f(x, y)dydx.$$

7. Find the volume under the surface $z = x^2 + y^2 + 1$ and above the half-disc $x^2 + y^2 \leq 4, \ y \geq 0$ in the xy plane. Use polar coordinates.

8. The acceleration of a particle is given by $\mathbf{a}(t) = \langle 1, -1, 0 \rangle$. Find the velocity and position of the particle as functions of time if at time $t = 0$ we have $\mathbf{v}(0) = \langle 0, 1, 0 \rangle$ and $\mathbf{r}(0) = \langle 1, 1, 0 \rangle$.

9. Find the absolute maximum and the absolute minimum of the function

$$f(x, y) = x^2 + y^2 - 4x + 2$$

on the region $\{(x, y) : 0 \leq x \leq 1, \quad -1 \leq y \leq 1\}$. Be sure to provide the coordinates of the points, and the absolute maximum and minimum values.

10. Using spherical coordinates, calculate the integral $\int\int\int_V (x^2+y^2+z^2)\,dxdydz$, where the region V is the half-ball: $\{x^2+y^2+z^2 \leq 1,\ y \leq 0\}$.

11. Calculate the integral

$$\int\int_D (2x - y)\, dA,$$

where the region D is bounded by the lines $2x - y = 1$, $2x - y = 2$, $x + 2y = 0$, $x + 2y = 1$. Use the change of variables $u = 2x - y$, $v = x + 2y$.

12. Calculate $curl\,\vec{F}$ and $div\,\vec{F}$ if $\vec{F} = (x^2 - y^2,\ xyz,\ x - z)$.

13. Check that the field $\vec{F} = (3x^2y - y,\ x^3 - x + 2y)$ is conservative by finding a potential f such that $\nabla f = \vec{F}$.

14. Find the mass of a wire that has the form of the quarter-circle $x^2 + y^2 = 1$, where $x \leq 0,\ y \leq 0$, if the density is given by $\rho(x, y) = 2 + x + y$.

15. Use Green's Theorem to find the integral

$$\oint_C yx^2\,dx - xy^2\,dy,$$

where C is the circle $x^2 + y^2 = 9$ and the integral is taken in a counter-clock-wise direction.