

MA 126 - 8C CALCULUS II

November 02, 2017

Name (Print last name first):

Student Signature:

TEST III

Closed book - Calculators and One Index Card are allowed!

PART I

Part I consists of 5 questions. Clearly write your answer (only) in the space provided after each question. Show your work to justify your answers. Very limited partial credit or none at all for this part of the test!

Each question is worth 8 points.

Question 1

Find a power series representation for the function $f(x) = \frac{x^5}{1-x}$. (Give your power series representation centered at $x = 0$.)

Answer:

Question 2

Determine whether the series $\sum_{n=1}^{\infty} \frac{1 + 5^{n-1}}{7^{n-1}}$ is convergent or divergent. Find its sum if it converges.

Answer:

Question 3

Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$ is convergent or divergent. Find its sum if it converges.

Answer:

Question 4

Determine whether the alternating series

$$\sum_{n=2}^{\infty} (-1)^n \frac{1}{n (\ln(n))^2}$$

is divergent, absolutely convergent, or conditionally convergent. (Be specific here!)

Answer:

Question 5

Given that the MacLaurin series of the function $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$, evaluate the indefinite integral $\int \frac{\sin(x)}{x} dx$ as an infinite series.

Answer:

PART II

Each problem is worth 15 points.

Part II consists of 4 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit - no credit for unsubstantiated answers!

Problem 1

Determine whether each of the following sequences is convergent or divergent. Find its exact (numerical) value if it converges. (Show your work!)

(a) $a_n = 2 - (-1)^n$

(b) $b_n = \tan(n\pi)$

(c) $c_n = \ln(5n - 1) - \ln(n)$

Problem 2

Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} (3x - 2)^n.$$

Be sure to check any endpoints that exist!

Problem 3

Express the function $f(x) = \frac{1}{x^2 + 3x + 2}$ as the sum of a power series by first using partial fractions for the function $f(x)$. Also find the actual interval of convergence of your series. (Show your work!)

Problem 4

The MacLaurin series of the natural exponential function e^x is given by $e^x = 1 + \sum_{n=1}^{\infty} \frac{x^n}{n!}$.

Answer all of the following questions.

(a) Write out a MacLaurin series representation for the function $f(x) = x^2e^{-x^2}$.

(b) Use the series in (a) to evaluate the (indefinite) integral $\int x^2e^{-x^2} dx$ as a power series.

(c) Use the series in (b) to write out a series representation for

$$\int_0^{0.5} x^2e^{-x^2} dx$$

(Do not compute and add up the terms of your series!)

(d) Find the minimum number of terms you need in the series in (c) to approximate $\int_0^{0.5} x^2e^{-x^2} dx$ with an error less than 10^{-3} ? (Show your work!)

SCRATCH PAPER

(Scratch paper will not be graded!)

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