MA 125   CALCULUS I
Final Exam, December 13, 2017

Name (Print last name first): ....

Show all your work, justify and simplify your answer!

No partial credit will be given for the answer only!

PART I

You must simplify your answer when possible but you don't need to compute numbers: $e^6 \sin(12/5) + 8$ is a fine answer.

All problems in Part I are 4 points each.

1. Use the **definition** of the derivative to show that the derivative of the function $y = f(x) = x^2 + 3x$ is $f'(x) = 2x + 3$.

2. Find the derivative $f'(x)$ if $f(x) = \sin(x) \ln(x)$.

3. Find the derivative $f'(x)$ if $f(x) = \cos(x^3 + 7x^2)$.

4. Find the derivative $f'(x)$ if $f(x) = \frac{x^2}{\sin(x)}$.
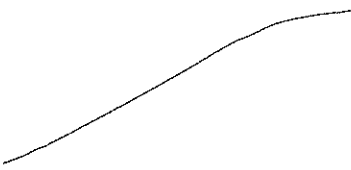
5. Find the anti-derivative $\int (x^3 + x)\sqrt{x}\, dx$.

6. Find the anti-derivative $\int \tan(x)\, dx$.

7. Find the anti-derivative $\int x^5(x^6 + 2)^{10}\, dx$.

8. Solve $\ln(5x + 2) = 4$.

9. If $F(x) = \int_0^x \sqrt[5]{t^7 + t^3}\, dt$, find $F'(x)$.

10. Find the linear approximation of the function $f(x) = \tan(x)$ at the point $a = \pi/4$ and use it to approximate the value of $\tan(.8)$. [**Do not use your claculator to find this value!**]
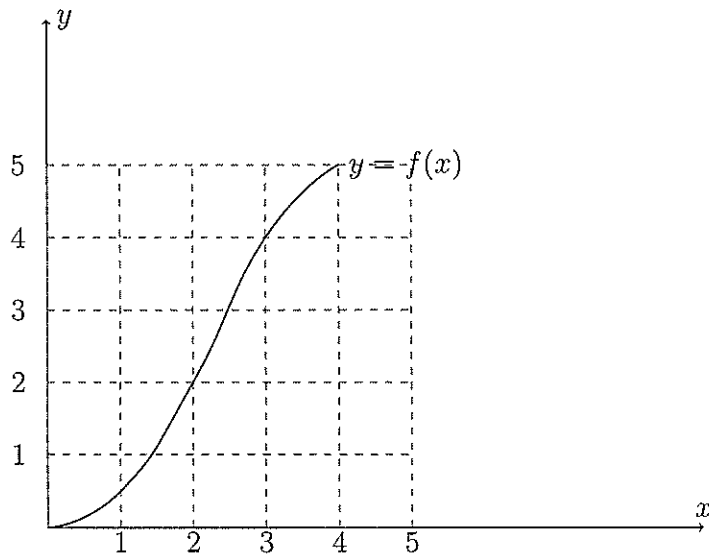
11. Approximate $\int_1^3 \sin(x^3)\,dx$ using a Riemann sum with $n = 3$ terms and the midpoint rule.

12. The velocity of a particle is given by $v(t) = \sin(t)\ (m/s)$.

   (a) Find the acceleration $a(1)$ of the particle,

   (b) How far does the particle travel in the first second?

13. Given the graph of the function $f(x)$ below answer the following questions.

$y = f(x)$

(a) Is $f(x)$ one-to-one? You must justify your answer !!

(b) Use the graph to approximate $f^{-1}(4)$.

(c) Use the graph to approximate $(f^{-1})'(4)$.

PART II

1. **9 points.** Find all local/absloute maxima/minima of the function $f(x) = (3x + 1)^4(1 - x)^6$ on the real line $(-\infty, \infty)$.

2. **9 points.** Show that the equation

$$f(x) = x^9 + 3x - 1 = 0$$

has at least one solution.

Use Newton's method with $x_1 = .2$ to compute the next approximate solution x_2.

3. **12 points.** Graph the function $y = f(x) = \dfrac{x^2}{x^2 - 4}$. Find x and y-intercepts, horizontal and vertical asymptotes, all critical numbers, intervals of in-/de-creasing, local/absolute max/min

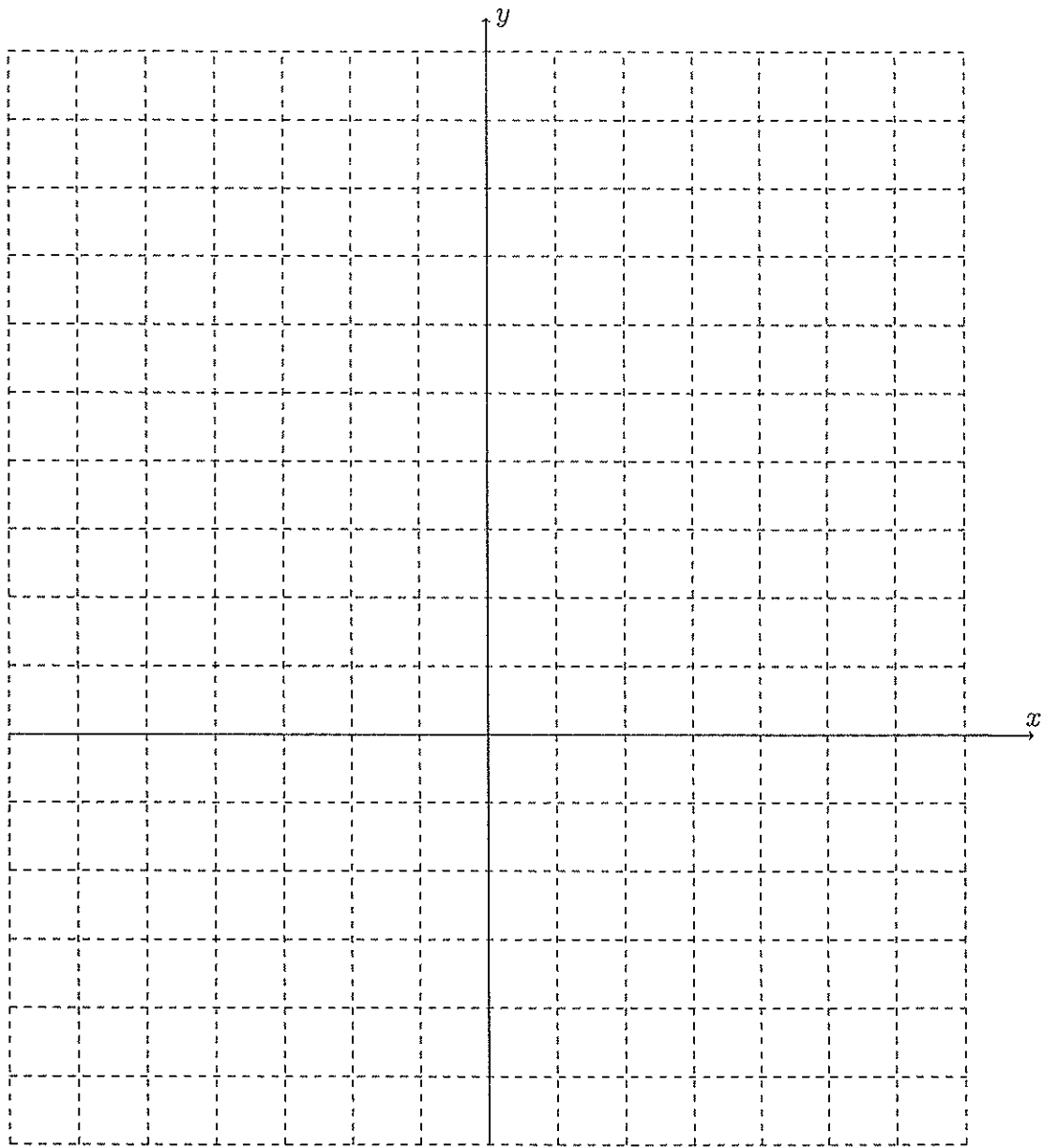**Draw your graph on the next page.** Find:

(a) x and y intercepts,

(b) vertical and horizontal asymptotes (if any),

(c) critical numbers,

(d) intervals where $f(x)$ is in/de-creasing, and local/absolute max/min.

4. **9 points.** Find the average value of the function $y = f(x) = \sin(x)$ on the interval $[\pi/4, 3\pi/4]$.

5. **9 points.** A soup company asks you to compute the dimensions of a soup can of Volume $V = 100\ cm^3$ of minimal cost if the top and bottom cost 2 $cents/cm^2$ and the side costs 1 $cent/cm^2$. The volume of a can of height h and radius r is $V = \pi r^2 h$, the area of the top and bottom is πr^2 and the area of the side is $2\pi r h$.

Scratch paper