

Instructor: _____ Name: _____

Final Exam
Calculus I; Fall 2011

Part I

Part I consists of 10 questions, each worth 5 points. Clearly show your work for each of the problems listed.

(1) Find y' if $y = \frac{x^2}{x^2+1}$

(2) Find y' if $y = [\sin(x)]^{10}$

(3) Evaluate $\lim_{x \rightarrow \infty} x e^{-x}$

(4) Find y' if $y = \cos(x^3 + x)$

(5) Find the critical points of $y = f(x) = (x + 1)^2(x - 1)^4$.

(6) Find all local maxima/minima of the function
 $y = f(x) = 3x^4 - 6x^2$. Make sure to state both x and y values.

- (7) Find the vertical and horizontal asymptotes of the function $y =$

$$f(x) = \frac{-2x^2+x+9}{(x+1)(x-2)}.$$

- (8) Find the interval(s) in the x -axis where $y = f(x) = x \ln(x)$ is **decreasing**.

- (9) Find the most general form for the **anti**-derivative of $y = x(\sqrt{x} + x^2)$.

- (10) If the horizontal side of a rectangle increases at a rate of 2 m/s and the vertical side decreases at a rate of 5 m/s , is the area of the rectangle increasing or decreasing at the moment that the horizontal side is 2 m and the vertical side is 6 m ? (**You must justify your answer!**).

Part II

Part II consists of 6 problems; the number of points for each part are indicated by [x pts]. You must show the relevant steps (as we did in class) and justify your answer to earn credit. Simplify your answer when possible.

- (1) [10 pts] Use implicit differentiation to find the derivative y' if $\sin(y) = xy$

- (2) [5 pts] Find the linearization of the function $y = f(x) = \sqrt{x}$ at $a = 9$.

(3) [**3 pts**] Use the linearization in problem 2 to estimate $\sqrt{9.2}$

(4) Given the function $y = f(x) = \frac{x^2-4}{x^2-1}$

(a) [**2 pts**] Find the x and y intercepts of the function.

(b) [**3 pts**] Find the vertical and horizontal asymptotes of the function.

(c) [**2 pts**] Find the open intervals where $f(x)$ is increasing and the open intervals where $f(x)$ is decreasing.

(d) [**2 pts**] Find the local maximum and local minimum values of $f(x)$ (if any). (Be sure to give the x and y coordinate of each of them).

(e) [**2 pts**] Find all open intervals where the graph of $f(x)$ is concave up and all open intervals where the graph is concave down.

(f) [**1 pts**] Find all points of inflection (if any). (Be sure to give the x and y coordinate of each point when possible.)

(g) [5 pts] Use the above information to graph the function below. Indicate all relevant information in the graph; in particular any **x,y-intercepts, local maxima/minima and point(s) of inflection.**

(5) [5 pts] If $y = f(x) = \arctan(x)$ find the absolute max and min on the interval $0 \leq x \leq 1$. (Include the appropriate y values of the maximum and minimum.)

- (6) [10 pts] Find the maximal area of a rectangle which is located in the upper half plane and under the graph of the function $y = f(x) = -x^2 + 6$. Note that the base of the rectangle will be contained in the x -axis. (Your answer should provide the area of the maximal rectangle!)

Scratch paper