

MA 126 - CALCULUS II
Friday, December 09, 2011

Name (Print last name first):.....

Signature:

Section: Instructor Name:

FINAL EXAM

Closed Book - No calculators!

PART I

Each question is worth 4 points.

Part I consists of 10 questions. Clearly write your answer (only) in the space provided after each question. You need not show your work for this part of the exam. No partial credit is awarded for this part of the exam! CHECK YOUR ANSWERS!

Question 1

Use vectors to decide whether the triangle with vertices $P(1, 0, 1)$, $Q(1, 0, 0)$, and $R(2, 3, 0)$ is right-angled.

Answer:

Question 2

Find the area of the parallelogram generated by the vectors $\mathbf{u} = \langle 1, 3, -1 \rangle$ and $\mathbf{v} = \langle 1, 0, -1 \rangle$.

Answer:

Question 3

Find the parametric equations of the line that passes through the point $P(1, 2, 1)$ and is parallel to the vector $\mathbf{v} = \langle 2, -5, 3 \rangle$.

Answer:

Question 4

Find an equation of the plane that passes through the point $P(1, -1, 0)$ and is perpendicular (normal) to the line with symmetric equations $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z-2}{3}$.

Answer:

Question 5

Use the Fundamental Theorem of Calculus to find the derivative of the function

$$g(x) = \int_0^x \arctan(t^2) dt.$$

Answer:

Question 6

Determine whether the improper integral is convergent or divergent. Evaluate the integral if it is convergent.

$$\int_1^{\infty} \frac{1}{x^{1.1}} dx$$

Answer:

Question 7

Find the area of the region bounded by the curve $y = x^2$ and the line $y = x$.

Answer:

Question 8

Evaluate the indefinite integral $\int \sin^7(x) \cos(x) dx$.

Answer:

Question 9

Evaluate the indefinite integral $\int \frac{x^2}{1+x^2} dx$.

Answer:

Question 10

Determine whether the alternating series $\sum_{n=1}^{\infty} (-1)^n \frac{n^2+6}{n^4+2}$ is divergent, absolutely convergent, or conditionally convergent.

Answer:

Problem 2

This problem has two separate questions. (Answer all the questions!)

- (a) Find the length of the arc of the circular helix with vector equation $\mathbf{r}(t) = \langle 3 \cos(t), 3 \sin(t), 4t \rangle$ when $-1 \leq t \leq 0$.

- (b) Determine whether the (improper) integral

$$\int_0^{\infty} x^2 e^{-x} dx$$

is convergent or divergent. Evaluate the integral if it is convergent.

Problem 3

Evaluate the following integrals (clearly show the techniques of integration you use):

(a) $\int \frac{1}{x\sqrt{\ln(x)}} dx$

(b) $\int x \cos(x) dx$

(c) $\int \frac{-x^2 + x + 2}{(x - 1)(x^2 + 1)} dx.$

Problem 4

This problem has two separate questions. (Answer all the questions!)

- (a) Find the **area** of the region enclosed by the parabola $x = y^2 - y$ and the parabola $x = y - y^2$.

- (b) The region enclosed by the curve $y = \sqrt{x}$ and the line $y = x$ and is rotated about the horizontal line $y = -1$. Find the **volume** of the solid obtained in this way.

Problem 5

This problem has two separate questions. (Answer all the questions!)

- (a) Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} (x + 5)^n.$$

Be sure to check any endpoints that exist!

- (b) Use the Maclaurin series of the function $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ to write out the Maclaurin series for the function $g(x) = \cos(x^2)$ and then write out the Maclaurin series expansion of $\int_0^1 \cos(x^2) dx$. (Do not compute and add up the terms of your series!)

Using the above information, find the minimum number of terms needed to approximate $\int_0^1 \cos(x^2) dx$ with an error less than 0.01?

DO NOT ENTER ANY PROBLEM SOLUTIONS OR WORK ON THIS PAGE.

Summary of scores on problems - for grading purposes only.

	Points
Part I	
Questions 1 – 10	
Part II	
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Total Test Score	

SCRATCH PAPER

(Scratch paper will not be graded)

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