

Calculus I Final Exam. April 2003. Name _____ Mathematically justify your answers (show work). Non-graphing, non-calculus calculators allowed. Simplify and complete all computations as much as possible. Circle answers. Remember that sloppiness (missing parentheses, illegible work, etc.) will cause lower scores.

PART I. Shorter problems (3 points each).

1. Find $g'(x)$ if
 - (A) $g(x) = 2x^3 + x^{-2}$
 - (B) $g(x) = \sin(5x^2 + 4)$
 - (C) $g(x) = x^3 \cos(x)$
 - (D) $g(x) = \frac{x}{1+x^2}$
 - (E) $g(x) = 5^x$
 - (F) $g(x) = \sin^2(x + x^3)$
2. Find $\lim_{x \rightarrow 5} \frac{3x^2 - 75}{x - 5}$
3. Find $\int_1^2 x^3 dx$
4. Find $\int \frac{3}{x} dx$
5. Find an equation of the line tangent to the curve $y = \frac{1}{x}$ at the point $(2, \frac{1}{2})$.

PART II. Longer problems (10 points each).

6. Let $g(x) = \frac{1}{x}$. Use the limit definition of derivative to show that $g'(a) = -\frac{1}{a^2}$.
7. Suppose a function y is defined implicitly by equation (*) $y^2 - 5xy - x^3 = 13$

- (A) Find $y' = \frac{dy}{dx}$.
- (B) Find the equation of the line tangent to the graph of equation (*) at the point $(1, -2)$.

8. Let $g(x) = x^4 - 4x^3$, Using calculus tools, identify all significant features of the graph of g , and sketch the graph.

9. A cylindrical container with volume 1000cm^3 is to be made. Find the radius and height that will make the total surface area minimal.

10. A tank has the shape of a cone with height 10 meters and radius 3 meters at the top. Water is pumped into the tank at the rate of 2 cubic meters per minute. How fast is the water rising when the water is 4 meters deep?

11. Find the largest and smallest values (and where they occur) for $f(x) = x^3 + \frac{3}{x}$ on the closed interval $[\frac{1}{2}, 2]$.

12. Evaluate the following:

- (A) $\int_1^9 \frac{1}{\sqrt{x}} dx$
- (B) $\int_0^2 (x^3 + 1)^2 dx$

Surprise Extra Credit (10 points)

Use Riemann sums and the limit definition of the integral to find $\int_0^3 x^2 dx$.

You may need the formulas $\sum_{k=1}^N k = \frac{N(N+1)}{2}$, $\sum_{k=1}^N k^2 = \frac{N(N+1)(2N+1)}{6}$.