MA 227-5D Spring 2003 Final Examination
Name ______________________________

1. Consider the surface given parametrically by the position vector

$$\mathbf{r}(u,v) = (u^2+v^2)\mathbf{i} + u\sin(v)\mathbf{j} + (u+1)\mathbf{k}.$$

Find the equation of the tangent plane to this surface at $(1+\pi^2/16, \sqrt{2}/2, \sqrt{2})$.

2. Let $z = f(x,y)$, $x = s+t$ and $y = s-t$. Show that

$$\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = \frac{\partial z}{\partial s}\frac{\partial z}{\partial t}.$$

3. Find the maximum value of $f(x,y,z) = x^2y^2z^2$ subject to the constraint

$$x^2+y^2+z^2 = 1.$$

4. Evaluate

$$\iint_D ye^x\,dA$$

where D is the triangular region having vertices $(0,0)$, $(2,4)$ and $(6,0)$.

5. Find the area of the surface having parametric equations

$$x = uv, y = u+v, z = u-v,$$

where $u^2+v^2 \leq 1$.

6. Evaluate

$$\iint_D \frac{x+2y}{\cos(x-y)}dA$$

where D is the region bounded by the lines $y = x$, $y = x-1$, $x+2y = 0$ and $x+2y = 2$.

7. Evaluate

$$\int_C \mathbf{F}\cdot d\mathbf{r},$$

where

$$\mathbf{F}(x,y,z) = y^2\cos(z)\mathbf{i} + 2xy\cos(z)\mathbf{j} - xy^2\sin(z)\mathbf{k}$$

and C has position vector

$$\mathbf{r}(t) = t^2\mathbf{i} + \sin(t)\mathbf{j} + t\mathbf{k},$$

where $0 \leq t \leq \pi$.

8. Evaluate

$$\iint_S (x\mathbf{i} - y\mathbf{j} + (x^2+y^2)z^2\mathbf{k})\cdot d\mathbf{S},$$

where S is the entire surface of the solid cylinder described by $x^2+y^2 \leq b^2$, $c \leq z \leq d$. Here c, d, and b are arbitrary positive numbers.