

Calculus III Test 3 March 25, 2003 NAME _____

STUDENT NUMBER: _____

No calculators, books, or notes allowed. Justify your answers by giving appropriate arguments and steps. Circle answers. All problems will be of equal value. Be sure to work the given problem; otherwise you will not receive credit.

1. Evaluate $\int_1^2 (xy - x^2) dx$

2. Evaluate $\int_1^4 \int_1^2 \frac{x}{y} + \frac{y}{x} dy dx$

3. Find the volume of the solid under the paraboloid $z = x^2 + y^2$ and above the region bounded by $y = x^2$ and $x = y^2$.

4. Let $f(x, y)$ be a function. Express the integral $\iint_D f(x, y) dA$ as an iterated integral in two ways, if the region D is the region bounded by $y = 3x$ and $y = x^2$.

5. Evaluate the iterated integral $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1} dx dy$.

6. Use polar coordinates to find the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 16$ and outside the cylinder $x^2 + y^2 = 4$.

7. Find the centroid of the region bounded by $y = x^2$ and $y = x^3$. (The centroid is the center of mass with mass density $\rho(x, y) = 1$ for all (x, y) .)

8. Use a triple integral to find the volume of the tetrahedron enclosed by the coordinate planes and the plane $2x + y + z = 4$.

9. Use spherical coordinates to find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 4$, above the xy -plane, and below the cone $z = \sqrt{x^2 + y^2}$.

Extra Credit: DO ONLY ONE (CIRCLE THE LETTER OF THE PROBLEM DONE):

A. Find the volume of the wedge cut from the cylinder $x^2 + y^2 = 1$ above the xy -plane and below the plane $y = z$.

B. Use integrals to show that the volume of a sphere of radius R is $\frac{4}{3}\pi R^3$.