

Mathematics 125 **FINAL EXAMINATION**

April 29, 2004

- Calculators are allowed *only* for numerical calculations, that is you may not graph functions on your calculator.
- There are a sheet of scratch paper attached at the end of the exam. Use it but do not tear them off the exam.
- Show your work; clearly write down each step in your calculations/reasonings. *No credit* is given to a correct numerical answer *without* any justification.

1. (20 pts) Differentiate the following functions.

a)

$$\frac{1 + x^2}{1 + x}$$

b)

$$\cos(\tan x)$$

c)

$$\frac{1}{(1+x^3)^{10}}$$

d)

$$10^{x^2}$$

Hint:  $10 = e^{\ln 10}$ .

**2.** (10 pts) Find values of the following limits

**a)**

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 - x - 2}.$$

**b)**

$$\lim_{x \rightarrow 0^-} \frac{|x|}{x}$$

**3.** (10 pts) Evaluate the following definite integrals.

**a)**

$$\int_1^4 \frac{1}{\sqrt{x}} dx$$

**b)**

$$\int_0^{2\pi} \cos x dx$$

4. (10 pts) A function  $f$  is defined as follows,

$$f(x) = \begin{cases} \frac{x^3 - 2x^2}{x - 2} & \text{if } x \neq 2 \\ A & \text{if } x = 2 \end{cases}$$

for some number  $A$ . a) Write down the **definition** that  $f$  is continuous at  $x = 2$ .

b) Choose a suitable value of  $A$  so that  $f$  becomes continuous at  $x = 2$ .

5. (10 pts) The table shows the population of Nepal (in millions) as of June 30 of the given year. Use a linear approximation to estimate the population at midyear in 1989.

t	1980	1985	1990	1995
P(t)	15.0	17.0	19.3	22.0

**6.** (20 pts) Consider  $f(x) = x^4 - 6x^2$ .

**a)** Find the intervals on which  $f$  is increasing and the intervals where  $f$  is decreasing.

**b)** Find the intervals where the graph of the function is concave up and the intervals where the graph of  $f$  is concave down.



c) Using the information obtained above, sketch a graph of  $f$ .

d) Find the local maximum and minimum values of  $f$ . Are they the absolute maximum or minimum?

7. (15 pts) A box with a square base and open top must have a volume of  $32\text{cm}^3$ . Find the dimension of the box (i.e. the height and the side length of the square) that minimizes the amount/area of the material to make the box.

**8.** (15 pts) A manufacturer produces a fabric with a fixed width. The quantity  $q$  of this fabric (measured in yards) that is sold is a function of the selling price  $p$  (in dollars per yard), which we write as  $q = f(p)$ . (For example  $f(15) = 15,000$  means if the price is set \$15 per yard, then the manufacturer sells 15,000 yards of the fabric.) **a)** Suppose we know  $f(20) = 10,000$  and  $f'(20) = -350$ . Find an approximate value of  $f(21)$ .

**b)** Note that the total revenue earned with selling price  $p$  is  $R(p) = p \cdot f(p)$ . Find the value of  $R'(20)$  and interpret your answer. In particular, as far as increasing the revenue, is raising the price from 20 a good move?

**9.** (10 pts) The velocity function  $v(t)$  of a car making a round-trip from Birmingham to Atlanta is given as below.

**a)** Estimate the distance between Birmingham and Atlanta using the graph of the velocity function.

**b)** Anniston is 50 miles away from Birmingham. When does the car pass by Anniston *on the way back*?