

MA 227: CALCULUS III
TEST #2, APRIL 1, 2004

Time allotted: 105 min.

Your name:

Sign here:

1. Determine the largest set on which the function $f(x, y) = \cot \frac{y}{x}$ is continuous.
10 points

2. Find all the second partial derivatives of $f(x, y) = x^5 - x^3y^3 + y^5$.
10 points

2

3. Find the differential of the function $w = \ln(x + y - z)$.

10 points

4. Let $u = xy + yz + zx$, $x = st$, $y = e^{st}$, $z = t^2$. Use the chain rule to find $\partial u/\partial s$ and $\partial u/\partial t$ when $s = 0$, $t = 1$.

10 points

5. Find all points (x, y) on the plane where the function $f(x, y) = x^3y^3$ has a nonzero gradient and the direction of the fastest increase is $\vec{i} + 2\vec{j}$.

10 points

6. Find the absolute minimum and maximum values of $f(x, y) = x^2y$ on the domain

$$D = \{(x, y) \mid x \geq 0, y \geq 0, x^2 + y^2 \leq 1\}.$$

10 points

4

7. Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y, z) = x^4 + y^4 + z^4$ subject to the constraint $x^2 + y^2 + z^2 = 1$.
10 points

8. Find the absolute extreme values of $f(x, y) = x^2 + 4y^2 - 3x$ on the disk $x^2 + y^2 \leq 1$.
10 points

9. Calculate the integral

$$\int_0^2 \int_0^1 \sqrt{x+y} dx dy.$$

10 points

10. Calculate

$$\iint_R \frac{xy^2}{x^2+1} dA$$

over the rectangle $[-1, 1] \times [0, 3]$.

10 points