MA 227: Calculus III
Final Test, May 4, 2004

Time allotted: 150 min.

Print your name:

Sign here:

1. The solid E in the first octant of space lies above the surface $z = 2(x^2 + y^2)$ and below the sphere $x^2 + y^2 + z^2 = 9/4$. Calculate its volume.

10 points

2. Evaluate

$$\iiint_E x dV,$$

where E lies between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 9$ in the first octant.

10 points

3. The lamina D is defined by the inequalities $0 \leq x \leq 1$, $0 \leq y \leq 1$, and its mass density function is given by $\rho(x, y) = x^2 + y^2$. Compute its mass and the center of mass.

10 points

4. The solid B lies inside the cylinder $x^2 + y^2 = 1$ and inside the ellipsoid $9x^2 + 9y^2 + z^2 = 36$. Calculate its volume.

10 points

5. Evaluate the integral by reversing the order of integration.

$$\int_0^9 \int_{y^{1/2}}^3 e^{x^3}\, dxdy.$$

10 points

6. Calculate the integral

$$\int_1^4 \int_1^2 (\frac{x^2}{y} + \frac{y^2}{x}) dydx.$$

10 points

7. Find the minimum and maximum values of the function $f(x, y, z) = yz + xy$ subject to the constraints $xy = 3$ and $y^2 + z^2 = 9$.

10 points

8. We know that x, y, and z are positive numbers the sum of which is equal to 1. Maximize the value of xy^2z^3.

10 points

9. Find the points on the ellipsoid $x^2+y^2+4z^2=1$ where the normal line is parallel to the line connecting the points $(3,-1,0)$ and $(5,2\sqrt{2}-1,1)$.

10 points

10. Let $z=y^2\tan x$, $x=t^2uv$, $y=u+tv^2$. Find $\partial z/\partial t$, $\partial z/\partial u$, and $\partial z/\partial v$ when $t=2$, $u=1$, $v=0$.

10 points