FINAL EXAM
MA227
APRIL 30, 2004

Name:

Note: Grades will be posted outside the classroom door by Wednesday, May 5. If you want your grade posted, write a number by your name above (and remember the number). Your grade will be posted by that number. If there is no number, then the grade will not be posted - it's your choice.

Closed Book. No calculators. Show your work.

1. (10 pts. each) Evaluate the following:
(a) $\frac{\partial}{\partial z}[y \ln(x - e^{-3z})]$

(b) The directional derivative of $f(x, y) = \sin(x)/\cos(y)$ in the direction of $\mathbf{u} = 3\mathbf{i} - 4\mathbf{j}$ at the point $x = y = \pi/4$.

Date: July 20, 2004.

(c) Compute the curl **F** for $\mathbf{F} := xy\mathbf{i} + yz\mathbf{j} + zx\mathbf{k}$.

(d) For **F** defined in (c), compute grad div **F**.

(e) Find $\frac{dF}{dt}|_{t=0}$ if $F(x,y) = e^{x^2-y}$ and $x = 1 + \sin 3t$, $y = 2\cos t$.

2. (10 pts.) Find the volume of the solid in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$) bounded by the surfaces $x + z = 3$ and $y + z = 3$.

3. (10 pts.) Calculate the integral

$$\int\int\int_E z^3\sqrt{x^2+y^2+z^2}dV$$

where E is the solid hemisphere (1/2 of a ball) with center at the origin, radius 2, that lies above the xy-plane.

4. (a) (10 pts.) Setup and calculate directly the line integral

$$\int_C x^2 y dx + 3xy dy$$

where C consists of the line from $(-1, 0)$ to $(1, 0)$ and the parabola $1 = x^2 + y$ from $(1, 0)$ to $(-1, 0)$.

(b) (10 pts.) Calculate the integral in part (a) using Green's Theorem.

5. (10 pts.) Use the Divergence Theorem to calculate the flux (i.e. the surface integral) of $\mathbf{F} := x^2y\mathbf{i} - yz\mathbf{j} + z^2x\mathbf{k}$ across the surface of the box with vertices $(\pm 1, \pm 2, \pm 2)$.

Test 3
MA227
April 7, 2004
Name:

Closed Book. No calculators. Show your work.
WORK ANY 5 OF THE FOLLOWING 6 PROBLEMS. The problems are worth 20 points each. INDICATE WHICH 5 PROBLEMS YOU HAVE CHOSEN TO WORK FOR CREDIT. (You may work the SIXTH problem for 5 pts. extra credit.)

1. Find the volume of the solid bounded by the surfaces $z = x^2 + y^2$ and $z = \sqrt{x^2 + y^2}$.

2. Find the area of the surface $z = y + \frac{2}{3}x^{3/2}$ that lies above the triangle in the plane $z = 0$ with vertices $(0, 0, 0)$, $(2, 0, 0)$, and $(0, 1, 0)$.

3. Find the volume of the solid bounded by above by $x^2 + y^2 + z^2 = 9$ and below by $\phi = \pi/4$ (where ϕ is a spherical coordinate, $0 \leq \theta \leq 2\pi$, and $0 \leq \rho < \infty$).

4. A wire forms the triangle in the plane with vertices $(0,0)$, $(1,1)$, and $(1,0)$. If the density of the wire is given by $\rho(x,y) = xy$, what is the mass?

5. Use Green's Theorem to evaluate the integral

$$\int_C (x^3 - y^3)dx + (x^3 + y^3)dy$$

where C is the boundary of the region between $x^2+y^2 = 1$ and $x^2+y^2 = 4$.

6. Evaluate the surface integral $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ for the vector field

$$\mathbf{F}(x, y, z) = \frac{x^2 + y^2}{z + 1}\mathbf{i} - \frac{x^2 + y^2}{z + 1}\mathbf{j} + 2z\mathbf{k}$$

where S is the part of the plane $x + y + z = 1$ in the first octant with downward orientation.

Test 2
MA227
March 1, 2004
Name:

Closed Book. No calculators. Show your work.

1. (a) (10 pts.) Find the rate of change of $f(x, y, z) = \cos(xy) + \sin(yz)$ at $(0, 0, \pi)$ in the direction of $2\mathbf{i} - 3\mathbf{j} + 2\sqrt{3}\mathbf{k}$.

   (b) (5 pts.) In which direction is f increasing most rapidly at $(0, 0, \pi)$? (Give the specific vector.)

2. (18 pts.)Let
$$f(x, y) = x^3 - 6xy + y^3.$$
Identify all local maxima, minima, and saddle points. Justify your answer.

3. (17 pts.) Find the maximum value of $f(x, y, z) = x + y$ on the surface of the ball of radius 2 and centered at the origin, i.e., $x^2 + y^2 + z^2 = 4$.

4. (The density of a thin plate represented by $x^2+y^2 = 12$ is given by $\rho(x, y) = 60y$. If we cut from the plate a region R represented by the portion of the plate above $y = x^2/\sqrt{2}$,
(a) (10 pts.) what is the mass of R?

(b) (5 pts.) Show how to calculate the center of mass of R. **For this part you may write the integrals without evaluating them.**

5. (18 pts.) Setup and evaluate an integral that will give the volume of the solid in the first octant bounded by $x + z = 2$ and $y + z = 2$

6. (17 pts.) Use polar coordinates to find the volume of the solid bounded by $f(x, y) = 8 - y^2 - x^2$ and $g(x, y) = y^2 + x^2$.

Extra Credit (4pts. No partial credit.) Evaluate

$$\int_0^{\frac{\pi}{2}} \int_x^{\frac{\pi}{2}} \frac{\cos y}{y} dydx.$$

Test 1
MA227
February 2, 2004
Name:

Closed Book. No calculators. Show your work.

1. A particle is at the point $(x(t), y(t), z(t))$ at time t where $x(t) = \cos(t-1)$, $y(t) = \sin(t-1)$, and $z(t) = 2t^{3/2}$.

   (a) (5pts.) What is the vector from the origin to the curve at any time t?

   (b) (10pts.) Give a vector that is tangent to the curve traced by the particle when it passes the point $(1, 0, 2)$ on the curve.

   (c) (10pts.) What is the length of the path traced by the particle as time passes from $t = 0$ to $t = 1$

2. (10pts.) If $z = f(x,y)$, **define**

$$\frac{\partial z}{\partial y}$$

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If $f(1,2) = 3$ and $f(1, 2\frac{1}{10}) = 5$, what is an approximate value for

$$\frac{\partial z}{\partial y}|_{(1,2)}?$$

3. (10pts.) Find the limit, if it exists, or show that the limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2}$$

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4. Calculate the following derivatives. Show your work.
(a.) (15pts.) For $f(x, y) = \cos(y)/(x^2 + y^2)$, find f_x and f_y.

(b.) (15pts.) Use the chain rule to find f_u if $f(x, y) = e^{x^2} y$ and $x = \ln(u^2 + v^2)$, $y = v/u$.

(c.) (10pts.) Assuming that $z = f(x, y)$, find $\frac{\partial z}{\partial y}$, if $x^2 + y^3 + z^2 - 3xyz = 4$.

5. Let $z = f(x, y) = \sqrt{x^2 + y^2}$.
(a.) (10pts.) Find the equation of the tangent plane at the point $(3, 4, 5)$ on the surface.

(b.) (5pts.) Use the tangent plane to approximate $f(3\frac{1}{3}, 4)$.