Calculus I

TEST 3

April 21$^{\text{st}}$, 2005

Name: ________________________________

- Show your work; clearly write down each step in your calculation/reasoning. *No credit* is given for a correct numerical answer without any justification.

1. *(10pts)* The height of a triangle is increasing at a rate of 2 cm/min while the area of the triangle is increasing at 3 cm^2/min. At what rate is the base of the triangle changing when the base is 15 cm and the area is 60 cm^2?

2. *(10pts)* Find the absolute maximum and absolute minimum values of $f(x) = (x^2 - 4x - 11)e^x$ on the interval $[-5, 0]$.

3. *(15pts)* Consider the function $f(x) = x^4 + 4x^3 + 17$.
(a) Where is the function increasing and where is the function decreasing? Write down your answers in interval notation.

(b) What are the local maxima and minima of $f(x)$?

(c) Where is the $f(x)$ concave up and where is the $f(x)$ concave down? Write down your answers in interval notation.

(d) What are the inflection point(s) of $f(x)$? Write down your answers in the form $(x, f(x))$.

(e) Use the above information to sketch the graph of $f(x)$.

4. Evaluate the following limits:

(a) *(5pts)*

$$\lim_{x \to -1} \frac{x+1}{x^3 - 1}$$

(b) *(5pts)*

$$\lim_{x \to 0^+} x^2 \ln x$$

(c) *(5pts)*

$$\lim_{x \to 1} \frac{1 - x + \ln x}{(x-1)^2}$$

5. (a) *(6pts)* Find the most general antiderivative of the function $f(x) = e^x + 2\cos(x) + \frac{3}{\sqrt{1-x^2}}$.

(b) *(6pts)* Find $g(x)$ when $g''(x) = 12x^2 + 2$, $g(1) = 2$ and $g'(0) = 3$.

6. *(10pts)* A box with a square base and open top must have a volume of 4,000 cm^3. Find the dimensions of the box that minimize the surface area of the box.

7. Evaluate the following integrals:
(a) *(7pts)*

$$\int_1^2 \frac{x^4 + x + 2}{x}\,dx$$

(b) *(7pts)*

$$\int_0^4 \sqrt{x}(x^2 + 2x)\,dx$$

8. The velocity function for a particle moving along a straight line is given by $v(t) = 2t - 6$.
(a) *(7pts)* Find the *displacement* of the particle during the time interval $1 \le t \le 5$.

(b) *(7pts)* Find the *distance traveled* by the particle during the time interval $1 \le t \le 5$.