

CALCULUS I

January 31, 2006

Name (Print last name first):

Student ID Number:

TEST I**PART I**

Part I consists of seven questions. Clearly write your answer (only) in the space provided. Do not show your work for this part of the test. No partial credit is awarded for this part of the test!

Each question is worth 4 points.

Question 1Given that $\lim_{x \rightarrow a} f(x) = 2$ and $\lim_{x \rightarrow a} g(x) = -3$, find

$$\lim_{x \rightarrow a} \frac{2f(x)}{g(x) - f(x)}.$$

Question 2

Evaluate

$$\lim_{h \rightarrow 0} \frac{(3+h)^{-1} - 3^{-1}}{h}$$

if the limit exists.

Question 3

In the theory of relativity, the length of an object with velocity v is

$$L = L_0 \sqrt{1 - v^2/c^2}$$

where L_0 is the length at rest and c is the speed of light. What happens to the length of the object when $v \rightarrow c^-$?

Question 4

In the theory of relativity, the mass of a particle with velocity v is

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where m_0 is the mass at rest and c is the speed of light. What happens to the mass of the particle when $v \rightarrow c^-$?

Question 5

$$\lim_{x \rightarrow \infty} (e^{-2x} \cos x) =$$

Question 6

Find the *slope* of the tangent line to the curve $y = \sqrt{x}$ at the point $(1, 1)$.

Question 7

$$\lim_{x \rightarrow \infty} \tan^{-1}(x^2 - x^4) =$$

PART II

Each problem is worth 8 points.

Part II consists of nine problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

Consider the function

$$f(x) = \begin{cases} 5x - 2 & \text{for } x < 4, \\ 4x + 3 & \text{for } x \geq 4. \end{cases}$$

(a) Evaluate

$$\lim_{x \rightarrow 4^-} f(x).$$

(b) Evaluate

$$\lim_{x \rightarrow 4^+} f(x).$$

(c) Is this function continuous at $x = 4$? (Justify your answer!)

Problem 2

Sketch the graph of an example of a function f that satisfies all of the following conditions.

$$\lim_{x \rightarrow 0^-} f(x) = 1, \quad \lim_{x \rightarrow 0^+} f(x) = -1, \quad \lim_{x \rightarrow 2^-} f(x) = 0,$$

$$\lim_{x \rightarrow 2^+} f(x) = 1, \quad f(2) = 1, \quad f(0) \text{ is undefined.}$$

Problem 3

Consider the function

$$f(x) = \frac{x^2 - 9}{2x^2 + 7x + 3}.$$

- (a) Is the function $f(x)$ continuous at the number $a = -3$ (justify your answer!)? If your answer is no, then what kind of discontinuity does the function $f(x)$ have at the number $a = -3$?

- (b) Evaluate the limit (if it exists)

$$\lim_{x \rightarrow -3} f(x).$$

- (c) If your answer in Part (a) is no, then what should the value of the function be at $a = -3$ in order to make it continuous there if possible? (That is, for the function to be continuous at $a = -3$, what should be the value of $f(-3)$, if need be?)

Problem 4

Evaluate the limit

$$\lim_{x \rightarrow 1} \arctan \left(\frac{\sqrt{3}x^2 - \sqrt{3}}{2x^2 - 2x} \right).$$

Problem 5

For what (numerical) value of the constant c is the function $f(x)$ continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + x & \text{if } x > 1, \\ x^3 - cx & \text{if } x \leq 1. \end{cases}$$

(Justify your answer!)

Problem 6

Consider the function

$$f(x) = x^3 - x + 1.$$

(a) Prove that the function has (at least) one real root. (Justify your answer!)

(b) Find an interval of length 1 that contains a root.

Problem 7

Evaluate (by showing your work)

$$\lim_{x \rightarrow \infty} \left(\sqrt{x^2 + x} - \sqrt{x^2 - 3x} \right)$$

if the limit exists.

Problem 8

Consider the function

$$f(x) = \frac{x^2 + x - 2}{x^2 + 3x + 2}.$$

(a) Find the horizontal asymptote of the curve $y = f(x)$. (Always show your work!)

(b) Find the vertical asymptote(s) of the curve $y = f(x)$. (Always show your work!)

Problem 9

The graph shows the position function of a car. Use the shape of the graph to explain your answers to the following questions.

(graph not included)

- (a) What was the initial velocity of the car?

- (b) Was the car going faster at B or at C ?

- (c) Was the car slowing down or speeding up at A and B ?

- (d) What happened between D and E ?