

CALCULUS I
February 28, 2006

Name (Print last name first):

Student ID Number:

TEST II

PART I

Part I consists of 7 questions. Clearly write and circle your answer (only) in the space provided after each question. Do not show your work for this part of the test. No partial credit is awarded on Part I problems.

Each question is worth 4 points.

Question 1

Let $f(x) = 3 - 4x + 2x^7$. Find the derivative $f'(x)$.

Question 2

Find $f'(x)$ when $f(x) = \sin x + \cos x$.

Question 3

Find the numerical value of the number a for which the function

$$f(x) = x^2 + ax + 5$$

has a *horizontal tangent line* at $x = 1$.

Question 4

Suppose that $f(x)$ and $g(x)$ are differentiable functions of x , and that $f(2) = -3$, $f'(2) = 2$, $g(2) = -2$, and $g'(2) = 4$. Find the numerical value of the derivative

$$\left(\frac{f}{g}\right)'(2).$$

Question 5

Find the derivative of the function $y = x \sin x$.

Question 6

Let $f(x) = e^x g(x)$, where $g(0) = -3$ and $g'(0) = 2$. Find the numerical value of the derivative $f'(0)$.

Question 7

Find the derivative of the function $y = \tan(\sqrt{x})$.

PART II

Each problem is worth 8 points.

Part II consists of 9 problems. You must show your relevant work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant step(s) will not get full credit.

Problem 1

Consider the function

$$f(x) = \frac{3\sqrt[3]{x^2}}{2} - 3.$$

- (a) Find the derivative $f'(x)$.

- (b) Is $f(x)$ differentiable at the number $x = 0$? (Justify your answer!)

- (c) Find the domain of $f'(x)$. (Write your answer in interval notation.)

Problem 2

Consider the curve

$$f(x) = 2x^3 + 3x^2 - 12x - 100.$$

- (a) Find the derivative $f'(x)$.

- (b) For what values of x does the curve $y = f(x)$ have a horizontal tangent line?

- (c) Find the coordinates of the points on the curve $y = f(x)$ where the tangent line is horizontal.

Problem 3

Consider the function

$$f(x) = 20 + 2e^x - 2x.$$

- (a) Find the derivative $f'(x)$.
- (b) On what open interval(s) is the function $f(x)$ increasing? (Justify your answer!)

Problem 4

Consider the function

$$f(x) = x^3 - 3x^2 + 5x.$$

- (a) Find the derivative $f'(x)$.
- (b) Find the second derivative $f''(x)$.
- (c) On what open interval(s) is the function concave up?
- (d) Does the function $f(x)$ have an inflection point? If your answer is yes, then give the coordinates of the inflection point.

Problem 7

Find $f'(x)$ when $f(x) = \frac{1-x}{1+x}$. (Simplify your answer!)

Problem 8

Find an *equation of the tangent line* to the curve

$$y = e^x \sin x - 1$$

at the point $(0, -1)$.

Problem 9

The first figure given below shows the graph of the *position* function of a car. Which one among the other graphs shows the *velocity* function and which one shows the *acceleration* function of the car. *Clearly* identify the corresponding graphs, and explain!

(figures not included)