MA 125 CALCULUS I SPRING 2006

May 05, 2006

FINAL EXAM

Name (Print last name first): ....

Student ID Number: ....

Instructor: .... Section: ....

PART I

Part I consists of 10 questions. Place your answer on the answer-line next to the question. Space is provided between questions for you to work each question (if you wish). No partial credit is awarded on Part I problems, only your entry on the answer line will be graded.

Each question is worth 4 points.

Question 1

Evaluate $\displaystyle\lim_{x\to-\infty}\frac{5x^3-x^2+3}{2x^3+x-9}$. Answer: ....

Question 2

Evaluate $\displaystyle\lim_{x\to 0}\frac{1-\cos x}{x^2}$. Answer: ....

Question 3

Find the derivative of the function $y = x \sin x$. Answer: ....

Question 4

Let $f(x) = \ln(1 - x^2)$. Find $f'(x)$. Answer: ....

Question 5

Find the derivative of the function $g(x) = \dfrac{x}{1+x}$. Answer: ....

Question 6

Let $f(x) = g(x) \ln x$, where $x > 0$. Find $f'(1)$ if $g(1) = -2$. Answer: ....

Question 7

Find the *slope* of the tangent line to the circle $x^2 + y^2 = 4$ at the point $(-\sqrt{2}, -\sqrt{2})$.

Answer: . . . .

Question 8

Find the open interval where the curve $y = \frac{1}{12}\, x^4 - \frac{1}{6}\, x^3$ is concave down.

Answer: . . . .

Question 9

Evaluate $\int (x + e^x)\, dx$

Answer: . . . .

Question 10

Evaluate $\int_0^{\pi/4} \sec^2 x\, dx$

Answer: . . . .

PART II

Each problem is worth 7.5 points.

Part II consists of 8 problems. You must show the relevant work on this part of the test to get full credit; that is, your solution must include enough detail to justify any conclusions your reach in answering the question. Partial credit may be awarded on Part II problems where it is warranted.

Problem 1

Consider the equation

$$x^2y^2 + xy - 2 = 0$$

in which y is implicitly defined as a function of x.

(a) Use implicit differentiation to find y'. Simplify your answer! (Factoring your answer might prove useful here!)

(b) Find an equation of the tangent line to the graph of this equation at the point $(1, 1)$.

(c) Find the x-coordinate of *each* point on the graph of this equation at which the slope of the tangent line is equal to -1. (You should find two x-coordinates in all!)

[Hint: First use Part (a) above to find an expression for y in terms of x. Then substitute this expression in the original equation to find an equation involving x only. Finally, solve for x by noticing that the equation you obtain leads to a 'quadratic' equation which you can factor...]

Problem 2

Consider the function

$$f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x.$$

(a) Find each open interval where $f(x)$ is increasing (you should find two intervals in all), and the open interval where it is decreasing.

(b) Find all local maximum and minimum values of the function $f(x)$.

(c) Find the open interval where $f(x)$ is concave down, and the open interval where it is concave up.

(d) Find the inflection *point* of $f(x)$.

(e) Sketch a graph of $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x$. [Clearly indicating the (relevant) items above on your graph.]

Problem 3

The edge of a cubic box was found to be 2 ft with a possible error in measurement of 0.03 ft.

Use *differentials* to estimate

(a) the maximum *possible error* in computing the volume of the box.

(b) the maximum possible *relative error* in computing the volume of the box.

(c) the maximum possible *percentage error* in computing the volume of the box.

[Hint: The volume of a cube is given by $V(x) = x^3$, where x is the length of each edge of a cube.]

Problem 4

A ball is propelled straight up into the air from the ground-level with an initial velocity of 32 ft/sec. It is known that the height of the ball after t seconds is given by

$$h(t) = 32t - 16t^2.$$

(a) Find the velocity $v(t)$ of the ball.

(b) Find the acceleration $a(t)$ of the ball.

(c) What is the maximum height the ball will reach.

(d) Assuming that the ball was thrown at time $t = 0$, how many seconds will elapse before the ball returns to the ground?

Problem 5

Suppose that 12 ft^2 of cardboard is available to make a box with a *square base* and *open top.* Find the *largest possible volume* of the box that can be made.

[Hint: The area of an open top box is given by $A = x^2 + 4xh$, where x is the length of each edge of the square base and h is the height of the box. In this case, the volume to be maximized is given by $V = x^2h$.]

Problem 6

A particle moves along a straight line with acceleration function

$$a(t) = 2 + 6t - 12t^2 \ \text{m/s}^2.$$

(a) Find the velocity $v(t)$ of the particle if it is known that its initial velocity $v(0) = 3$ m/s.

(b) Find the position $s(t)$ of the particle if it is known that its initial position $s(0) = 10$ m.

Problem 7

Evaluate the following definite integrals.

(a) $\displaystyle\int_0^1 \left(4x^3 - 6x + \frac{2}{\sqrt{1-x^2}}\right)\,dx.$

(Hint: Recall that $\sin^{-1} 1 = \dfrac{\pi}{2}$.)

(b) $\displaystyle\int_1^e \left(1 + \frac{1}{x}\right)\,dx.$

Problem 8

Calculate the area of the region bounded by the x-axis and the graph of the curve $y = x^3 - 2x^2 + x$ for $-1 \leq x \leq 1$.

[Hint: An *appropriate* use of definite integrals might prove useful here! Note that the function $y = x^3 - 2x^2 + x$ has x-intercepts at the numbers $x = 0$ and $x = 1$, and that it is negative (!) on the interval $-1 \leq x < 0$ and positive on the interval $0 < x < 1$.]

Summary of scores on problems - for grading purposes only. Do not enter any problem solutions or work on this page.

	Points
Part I - Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Question 6	
Question 7	
Question 8	
Question 9	
Question 10	
Part II - Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7	
Problem 8	
Total Test Score	