MA 125: Calculus I, Spring 2008
FINAL EXAM
April 25, 2008

Your name (Print, last name first):

Your signature:

Instructor and section:

PART I. Consists of ten questions, 4 points each. Space is provided between questions for you to work each question. No partial credit is awarded here, only your BOXED ANSWER will be graded. Box your answer, please.

1. Evaluate

$$\lim_{x\to 4} \frac{x^2-16}{x-4}.$$

2. Evaluate

$$\lim_{x\to 0} \frac{e^{3x}-3x-1}{x^2}.$$

3. For what numerical value of a is the function

$$f(x) = \begin{cases} 2x^2 + 3a - 2 \text{ if } x \leq 5 \\ ax \text{ if } x > 5 \end{cases}$$

continuous for all x?

4. Find the value of x for which the curve $y = 3x - 2\ln x$ has a horizontal tangent.

5. Find an equation of the tangent line to the curve $y = x^2 - 2x$ at the point $(3, 3)$.

6. Let $h(x) = f(g(x))$, where $g'(2) = -2$, $g(2) = 5$, and $f'(5) = 4$. Find $h'(2)$.

7. Find all critical numbers of the function $f(x) = x^{6/5} - 2x^{1/5}$.

8. Find the open interval(s) on which the function $f(x) = \dfrac{e^x}{x+2}$ is increasing.

9. Find all inflection points of the curve $y = x^4 - 8x^2$. (Be sure to give the x and y coordinates of each point.)

10. Find the most general antiderivative of the function $f(x) = e^{-x} + \cos x + \sec x \tan x$ on the interval $(-\pi/2, \pi/2)$.

PART II. Six partial credit problems, each for 10 points. You MUST show your relevant work on this part of the test to get full credit, that is, your solution MUST include enough detail to justify any conclusions you reach in answering the question. Partial credits MAY be awarded on Part II problems where they are warranted.

Problem 1. Consider the implicit equation $5x^2 + 3xy + y^2 = 15$.

(a) Use implicit differentiation to find y'.

(b) Is the curve $5x^2 + 3xy + y^2 = 15$ rising or falling at the point $(1, 2)$? (Justify your answer!)

(c) Find an equation of the tangent line to the curve $5x^2 + 3xy + y^2 = 15$ at the point $(1, 2)$.

Problem 2. Consider the function $f(x) = x^3 - 3x + 2$.

(a) Find each open interval on which $f(x)$ is increasing, and each open interval on which $f(x)$ is decreasing.

(b) Find all local maximum and minimum values of f, and the x coordinates where these values are attained.

(c) Find the open interval(s) where f is concave down, and the open interval(s) where it is concave up.

(d) Find the inflection point on the curve $y = f(x)$. (Be sure to give the x and y coordinates!)

(e) Sketch the graph of $y = x^3 - 3x + 2$. (Clearly indicating on the graph the relevant items listed above.)

Problem 3. Consider the function $f(x) = \sqrt[6]{x}$.

(a) Find the linear approximation (linearization) of $f(x)$ at $a = 1$.

(b) Use the above linearization to find an approximation of $\sqrt[6]{1.1}$.

(c) Another way to find an approximation of $\sqrt[6]{1.1}$ is to use Newton's method to find a root of the equation $x^6 - 1.1 = 0$. Use Newton's method with initial approximation $x_1 = 1$ to find x_2, the second approximation of the requested root.

Problem 4. A stone is thrown upward from the ground at time $t = 0$. It is known that its height (in feet) after t seconds is given by $s(t) = 192t - 16t^2$. Answer the following questions.

(a) Find the velocity $v(t)$ after t seconds.

(b) Find the acceleration $a(t)$ after t seconds.

(c) What is the maximum height the stone will reach?

(d) How many seconds will elapse before the stone strikes the ground again? Also, determine the impact velocity.

Problem 5. A farmer wants to fence off a rectangular field that borders a straight river (with no fancing along the river). The field is to have an area of $5,000\ m^2$. What are the dimensions of the field that use the least amount of fencing?

Problem 6. Use antiderivatives to answer the following questions.

(a) Find $f(x)$ for $x > 0$ if it is known that $f'(x) = \dfrac{x^2 - 3x + 2}{x}$ and that $f(1) = 1$.

(b) Find the most general antiderivative $F(x)$ of the function

$$\frac{1}{\sqrt{1-x^2}},$$

and the evaluate $F(1/2) - F(0)$.