

MA 126-6D , CALCULUS II

February 6, 2008

Name (Print last name first):

Student Signature:

TEST I

Closed book test – No Calculators are permitted!

PART I - Basic Skills

Each question is worth 5 points.

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You do not need to show your work for this part of the test. No partial credit is awarded for this part of the test!

Question 1

Find an equation of a sphere that passes through the point $P(2, 4, 5)$ and has center $C(1, 2, 3)$.

Answer:

Question 2

Find the dot product of two vectors \mathbf{u} and \mathbf{v} if $|\mathbf{u}| = 6$, $|\mathbf{v}| = \frac{1}{3}$, and the angle between them is $\frac{\pi}{4}$.

Answer:

Question 3

For what (numerical) values of b are the vectors $\langle 3, 2, b \rangle$ and $\langle 2b, 4, b \rangle$ orthogonal?

Answer:

Question 4

Find a vector which is orthogonal to both vectors $\langle -1, 0, 2 \rangle$ and $\langle 1, 2, 3 \rangle$.

Answer:

Question 5

Find parametric equations of the line that passes through the point $P(1, 0, -1)$ and is parallel to the vector $\mathbf{v} = \langle 3, 2, 1 \rangle$.

Answer:

Question 6

Find an equation of the plane through the points $P(0, 1, 1)$, $Q(1, 0, 1)$, and $R(1, 1, 0)$.

Answer:

PART II - Problem Solving skills

Each problem is worth 14 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

This problem has two separate questions (a) and (b). Answer each question.

- (a) A constant force $\mathbf{F} = 3\mathbf{i} + 5\mathbf{j} + 10\mathbf{k}$ moves an object along the line with displacement vector $\mathbf{D} = -5\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$. Find the work done if the (magnitude of the) displacement is measured in feet and the (magnitude of the) force is measured in pounds.
- (b) A woman walks due north on the deck of a ship at 2 mi/h. The ship is moving east at a speed of $\sqrt{12}$ mi/h.
- (1) Find the speed of the woman relative to the surface of the water. (That is the magnitude of the resultant of the velocity vectors.)
 - (2) Find the direction of the woman relative to the surface of the water. (That is the direction of the resultant velocity. Express your answer as an angle with respect to specific direction(s) north, south, east, or west.)

Problem 2

This problem has two separate questions (a) and (b). Answer each question.

(a) Find the area of the triangle with vertices $P(7, 8, 7)$, $Q(7, 7, 7)$, and $R(7, 7, 8)$.

(b) Find the volume of the box determined by the vectors $\mathbf{a} = 6\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, $\mathbf{b} = \mathbf{j} + 2\mathbf{k}$, and $\mathbf{c} = 4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$.

Problem 3

(a) Find the parametric equations for the line passing through the points $P(2, 0, 1)$ and $Q(-1, 3, 2)$.

(b) Determine the coordinates of the point where the line obtained in (a) intersects the xz -plane.

Problem 4

This problem has two separate questions (a) and (b). Answer each question.

- (a) Find symmetric equations of the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$.

- (b) Find an equation of the plane that passes through the point $P(4, 0, -3)$ and is perpendicular to the vector $\mathbf{n} = \langle 2, 4, 8 \rangle$.

Problem 5

An particle is traveling along the space-curve

$$\mathbf{r}(t) = \left\langle \frac{1}{1+t}, t^2 + 5, te^{-t} \right\rangle$$

when the time $t > 0$.

(a) Evaluate $\lim_{t \rightarrow 0} \mathbf{r}(t)$.

(b) Determine the velocity-vector of the particle at each time t .

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