MA 227, Calculus - III. THE FINAL EXAM Friday, May 1, 2009.

Student's Name ______________________________

. *(Please, print)*

GIVE REASONS FOR YOUR ANSWERS!

CODE:

I. (6%) Find the length of the curve:

$$\vec{r}(t) = (4\sin t, 3t, 4\cos t), \quad -10 \le t \le 10.$$

II. (4%+6%) The position function of a particle is given by the formula $\vec{r}(t) = (t, t^2, 3t)$.

a) Find the velocity, acceleration and the speed of the particle.

b) Find tangential and normal components of the acceleration vector.

III. (6%+9%)

a) Find the curvature of the curve $\vec{r}(t) = (t^2, \frac{2}{3}t^3, t)$ at the point $(1, \frac{2}{3}, 1)$.

b) Find vectors $\vec{T}, \vec{N}, \vec{B}$ for this curve at the given point.

IV. (4%+4%)

a) Find the equation of the tangent plane to the surface $z = x^3 + y^2 - 4xy$ at the point $(2, 1, 1)$.

b) Find the linearization of the function $f(x, y) = x^3 + y^2 - 4xy$ at the point $(2, 1)$. Use the linearization to calculate $f(2.001, 0.098)$.

V (6%) **Use the chain rule** to find $\frac{\partial u}{\partial t}$, $\frac{\partial u}{\partial s}$:

$$u(x, y, z) = xyz,$$

$$x = s^2 t, \quad y = e^{st}, \quad z = t^2.$$

VI. (4%+6%)
a) Find the gradient of the function $f(x, y, z) = x^2y + z^3$ at the point $P(1, -2, 1)$.
b) Find the derivative of this function in the direction of the vector $\vec{u} = \frac{1}{\sqrt{3}}(1, -1, 1)$ at the same point P.

VII (6%) Find the maximum value of the directional derivative of the function

$$f(x, y, z) = y + \frac{x}{z},$$

at the point $(3, 4, -1)$ and the direction in which it occurs.

VIII. (6%) Find local maxima (if any), local minima (if any) and saddle points (if any) of the function:

$$f(x, y) = x^3 - 6xy + 8y^3.$$

IX. (7%) Find

$$\int_D \int (y + x) dA,$$

where D is a triangle region with vertices $(0,0)$, $(2,0)$, $(1,1)$.

X. (6%) Find the volume of the solid bounded with the cylinder $x^2 + y^2 = 1$, the plane $z = 0$ and the paraboloid $z = 4 - x^2 - y^2$.

XI (6 %). Use spherical coordinates to find the volume of a sphere of radius a.

XII (6%). Find

$$\int_C \sqrt{x^2 + y^2}\, ds,$$

where C is the circle of the radius 4 centered at the origin.

XIII(8%). Find $\int_C \vec{F} \cdot d\vec{r}$,

$$F(x,y,z) = x^2\vec{i} + xy\vec{j} + z^2\vec{k},$$

$$\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}, \quad 0 \le t \le 1.$$

XIV **Extra credit** (10%). Use Green's Theorem to evaluate the line integral

$$\int_C x^2y dx + xy^5 dy,$$

where C is the positively oriented square with vertices at $(\pm 1, \pm 1)$.