EGR 265, Math Tools for Engineering Problem Solving
May 2, 2011, 10:45am to 1:15pm

Name (Print last name first): ....

Student ID Number: ....

Final Exam

$P1$:	$P2$:
$P3$:	$P4$:
$P5$:	$P6$:
$P7$:	$P8$:
$P9$:	$P10$:
$P11$:	

Problem 1 (8 points)

Find an explicit solution of the initial value problem

$$e^x y \frac{dy}{dx} = 1, \quad y(0) = 2.$$

Problem 2 (8 points)

Note: In this problem write your answers in terms of natural logarithms, which do not need to be evaluated.

Iodine-131 has a half-life of 8 days.
(a) Find its decay rate k.
(b) If the initial amount of Iodine-131 is 1 gram, how much of it is left after 2 days?
(c) How long does it take for Iodine-131 to decay to 10 percent of its original amount?

Problem 3 (14 points)

Consider the second order differential equation

$$y'' - 10y' + 25y = 30x + 3. \tag{1}$$

(a) Find the general solution of the homogeneous equation corresponding to (1).
(b) Find a particular solution of the inhomogeneous equation (1).
(c) Solve the initial value problem given by (1) and initial conditions $y(0) = 1$, $y'(0) = 3$.

Problem 4 (12 points)

A mass of 12 kg stretches a spring by 40 cm. Include the correct units in all your answers below.
(a) Find the spring constant k, assuming that $g = 10$ m/s^2.
(b) Find the equation of motion of the mass if it is released 30 cm below the equilibrium position at a upward velocity of 2 m/s (choose the positive x-axis to be oriented downward).
(c) Find the amplitude at which the mass oscillates.

Problem 5 (10 points)

(a) Find the gradient of $f(x, y) = x \ln(x^2 + y)$.
(b) Evaluate the directional derivative of $f(x, y)$ at the point $P(2, -3)$ in the direction of the vector $\mathbf{i} - 2\mathbf{j}$.
(c) Find a unit vector in the direction of steepest decrease of $f(x, y)$ at the point $P(2, -3)$. Also find the rate of decrease in this direction.

Problem 6 (8 points)

Find an equation of the tangent plane to the level surface $\sin(xyz) - x - 2y - 3z = 0$ through the point $(2, -1, 0)$.

Problem 7 (8 points)

Find the work done by the force field

$$\mathbf{F}(x, y) = e^x\mathbf{i} + xy\mathbf{j}$$

along the curve parameterized by $x = t^2$, $y = t^3$, $0 \leq t \leq 1$.

Problem 8 (12 points)

(a) Verify that the force field $F(x, y) = (2x \cos y)\mathbf{i} + (\cos y - x^2 \sin y)\mathbf{j}$ is conservative.
(b) Find a potential function $\phi(x, y)$ for $F(x, y)$.
(c) Find the work done by the force field $F(x, y)$ along the curve $x = t^2 + \frac{1}{2}$, $y = t$, $0 \leq t \leq \frac{\pi}{2}$.

Problem 9 (10 points)

Find the double integral of the function $f(x, y) = x^3y^2$ over the triangle in the xy-plane with vertices $(0, 0)$, $(1, -1)$ and $(1, 1)$.

Problem 10 (10 points)

A lamina of density $\rho(x, y) = 1 + x + y$ occupies the half disk R that lies above the x-axis within the circle $r = 2$. Find the mass of the lamina.

Problem 11 (6 points Bonus)

The function $f(x, y) = 2x^2 + y^2 - 3x$ is defined on the unit disk $x^2 + y^2 \leq 1$ and takes its maximum value and minimum value at two different points of the disk. Find the two points and the maximum and minimum values.