

FINAL EXAM

Name (Print last name first):.....

Instructor: Section:

Part 1

Part I consists of 10 questions each worth 5 points. Place your answer on the answer-line next to the question. Clearly show your work for each of the problems listed. No calculators!

1. Find the equation of the tangent line to $y = \cos(2x)$ when $x = \frac{\pi}{4}$. _____

2. Find the derivative of $y = (\tan(2x))^9$. _____

3. Find the most general antiderivative of $f(x) = x - \frac{2}{x} + \sqrt{x} + \sin x$. _____

2

4. If $f'(x) = \frac{2}{\sqrt{1-x^2}}$ and $f(\frac{1}{2}) = 1$, find $f(x)$.

5. Evaluate $\lim_{x \rightarrow 0^+} x \ln x$.

6. Find the x -coordinate of each critical number of $f(x) = x + \frac{3}{2}x^{2/3}$.

7. Find the value of x for which the graph of $y = \frac{\ln x}{x}$ has a horizontal tangent. _____

8. Use Newton's Method with initial approximation $x_1 = -2$ to find x_2 , a second approximation to the root of the equation $x^3 + 7 = 0$. _____

9. Use calculus to find the x -coordinate(s) of the inflection point(s) of $f(x) = xe^{-x}$. _____

10. Find the linearization $L(x)$ of $y = \sqrt{x}$ at $x = 16$. _____

PART II

Each problem is worth 8 points except #6 which is worth 10 points. No calculators!

Part II consists of 6 problems. You must show the relevant work on this part of the test to get full credit; that is, your solution must include enough detail to justify any conclusions you reach in answering the question. Partial credit may be awarded on Part II problems where it is warranted.

1. Assume that the equation $y^2 - 2xy + x^2 = 2x + 4$ defines y implicitly as a function of x .

a. Use implicit differentiation to find $\frac{dy}{dx}$. Simplify your answer.

b. Find the line normal to the curve $y^2 - 2xy + x^2 = 2x + 4$ at the point $(6, 2)$.

5

2. A rock thrown vertically upward from the surface of the moon at a velocity of 24 m/sec reaches a height of $s = 24t - \left(\frac{4}{5}\right)t^2$ meters in t sec. Use calculus to answer the following questions.

a. Find the velocity $v(t)$ and acceleration $a(t)$ of the rock after t seconds.

b. How high does the rock go?

c. How long is the rock aloft?

d. With what impact velocity does the rock hit the ground?

3. A rectangular box with a square base and an *open* top is to have a volume of 72 cubic inches. Find the dimensions of the box that minimize the surface area..

4. Use calculus to determine the absolute maximum and minimum values of $f(x) = 2x^3 - 3x^2 - 12x + 5$ on the interval $[-2, 4]$.

5. Mail truck A is heading directly west away from the Opelika Post Office at 36 mph. At the same time, mail truck B is heading directly south toward the post office at 60 mph. How fast is the distance between the two trucks changing when A is 4 miles and B is 3 miles from the post office?

6. Let $f(x) = x^5 - 5x^4$. In (a)—(d) below, *show how you use calculus* to

a. find the intervals on which f is increasing and decreasing.

b. find the local maximum and minimum values of f .

c. find the intervals of concavity and the inflection points.

d. Use the information found above to draw the graph of $y = f(x)$. Label all maximum and minimum points, intercepts, and inflection points.

SCRATCH SHEET