EGR 265, Math Tools for Engineering Problem Solving
April 22, 2015, 10:45am to 1:15pm

Name (Print last name first): ....

Student ID Number: ....

Final Exam

Problem 1	
Problem 2	
Problem 3	
Problem 4 (incl. Bonus)	
Problem 5	
Problem 6	
Problem 7	
Problem 8	
Problem 9	
Problem 10	
Total (out of 100 + 6 Bonus)	

Problem 1 (8 points)

Find an explicit solution of the initial value problem

$$y' + 3y = 6e^{-x}, \quad y(0) = 5.$$

Problem 2 (10 points)

A population of bacteria grows proportional to the number of bacteria present at time t. Suppose that the initial population is 100 and that the population after 2 hours has grown to 150. (Note: Write your answers to parts (b) and (c) in terms of natural logarithms, which do not need to be evaluated.)

(a) With $x(t)$ denoting the size of the population at time t, solve the Malhusian model $x'(t) = kx(t)$, $x(0) = 100$, as a linear or separable DE to find the formula for $x(t)$. Here k is still unknown.

(b) Find the growth rate k of the population.

(c) How long does it take for the population to double in size?

Problem 3 (12 points)

Find the general solution of the second order differential equation

$$y'' - 3y' + 2y = -\sin x. \tag{1}$$

Problem 4 (10 points + 6 points bonus)

A mass of 4 kg stretches an undamped spring by 10 cm. For simplicity, assume that $g = 10$ m/s^2.

(a) Find the spring constant k, including its correct unit. Also find the angular frequency ω of the spring-mass system.

(b) Set up the second order differential equation which governs the motion of the spring-mass system, choosing the x-axis to be oriented downwards. Find the general solution of this equation.

(c) Find the particular solution of the equation if the mass is released from 1 meter above the equilibrium position at a downward velocity of 50 cm/s.

(d) (Bonus) In the DE for undamped free motion which you found in (b) add an exterior driving force of the form

$$F(t) = 5\sin(10t)$$

or

$$F(t) = 5\sin(11t).$$

For both cases, without solving the DE, sketch what type of graph you expect for the solution $x(t)$ and name the physical phenomenon which is seen.

Problem 5 (10 points)

(a) Find the gradient of $f(x, y) = ye^{3x}$.

(b) Evaluate the directional derivative of $f(x, y)$ at the point with coordinates $(0, 1)$ in the direction of the vector from $(0, 1)$ to $(1, 3)$.

(c) Find a unit vector in the direction of steepest decrease of $f(x, y)$ at the point $(0, 1)$.

Problem 6 (10 points)

(a) Determine the equation of the tangent plane to the level surface $2x^2 + y^2 + 5z^2 = 11$ through the point $(1, 2, 1)$.

(b) Find the parametric equations for the normal line through $2x^2 + y^2 + 5z^2 = 11$ at the point $(1, 2, 1)$

Problem 7 (8 points)

Find the line integral

$$\int_C x\,ds,$$

where C is the graph of the function $y = \frac{1}{2}x^2$, $0 \le x \le 2$.

Problem 8 (12 points)

(a) Verify that the force field $\mathbf{F}(x, y) = (y^2 - 4xy)\mathbf{i} + (2xy - 2x^2 + 1)\mathbf{j}$ is conservative.

(b) Find a potential function $\phi(x, y)$ for $\mathbf{F}(x, y)$.

(c) Find the work done by the force field $\mathbf{F}(x, y)$ along the curve parameterized by $x = \cos(2t)$, $y = \sin(t)$, $0 \leq t \leq \pi/2$.

Problem 9 (10 points)

(a) Find the double integral $\iint_R x^2\, dA$, where R is the region in the xy-plane between the x-axis and the graph of $y = 1 - x^2$.

(b) What is the physical meaning of this integral? Do not just state the physical name of this quantity, but explain what is measured by it.

Problem 10 (10 points)

Express the function

$$f(x,y) = \frac{1}{(x^2+y^2)^2}$$

in polar coordinates and find its double integral over the washer shaped region between the two disks of radius 1 and 2, both centered at the origin.