MA 125 CT, CALCULUS I

February 25, 2015

Name (Print last name first): ....

Show all your work and justify your answer!

No partial credit will be given for the answer only!

PART I

You must simplify your answer when possible.
All problems in Part I are 10 points each.

1. Find the derivative of the function $y = f(x) = x^2 \sin(x^3)$.

2. Find the derivative of $f(x) = (x^3 + x)^7$.

3. Find the absolute maximum and minimum of the function $y = f(x) = (x-3)^2(x+1)$ on the interval $[0, 1]$.

4. Verify that the conditions of the Mean Value Theorem hold. Next find the number c which satisfies the conclusion of the Mean Value Theorem for the function $y = f(x) = x^2$ on the interval $[1, 2]$.

5. Find all critical numbers of the function $y = f(x) = \sqrt{x^3 + x}$ on $[0, \infty)$.

6. Suppose that the **derivative** of a function $y = f(x)$ is given:

$f'(x) = (x+1)(5-x)$.

(a) Find the x-coordinates of all local max/min of the function $y = f(x)$.

(b) At which x is the function $y = f(x)$ most rapidly increasing?

PART II

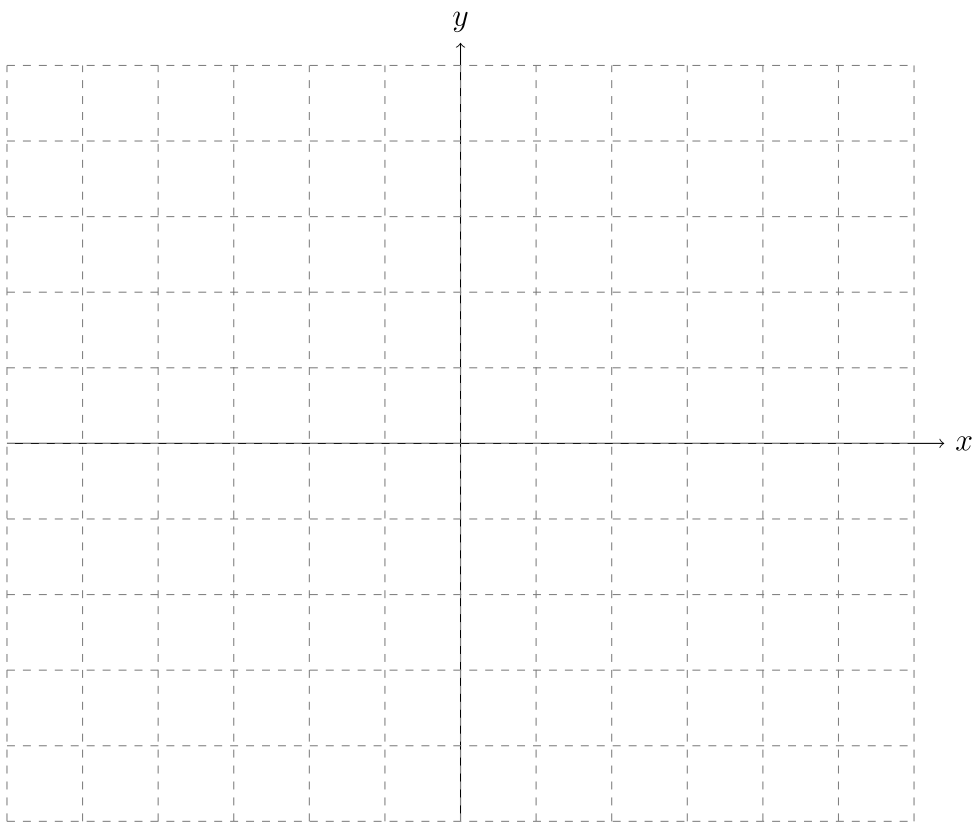
7. [**10 points**] Suppose you are asked to design a soup can of volume 1 dm^3 of minimal cost if the cost of the top and bottom is $5/m^2$, and the cost of the sides is $3/m^2$. Your answer should specify the dimensions of the can!

   You may use that the volume of a can of radius r and height h is $V = \pi r^2 h$ while the surface are of the side is $2\pi r h$ and of the top/bottom is πr^2.

8. [**20 points**] **Use calculus** to graph the function $y = f(x) = \dfrac{x^2 - 1}{x^3}$. Indicate

- x and y intercepts,
- vertical and horizontal asymptotes (if any),
- in/de-creasing; local/absolute max/min (if any).

You must show work to justify your graph and conclusions. You can use decimal numbers to plot points (but mark them with exact values).



9. This question has two parts.

   (a) [**6 points**] Find the linearization of $f(x) = \sqrt{x}$ at $a = 100$

   (b) [**4 points**] Use this linearization to find the approximate value of $\sqrt{101}$.