DEPARTMENT OF MATHEMATICS UAB
CALCULUS II FINAL EXAMINATION
WEDNESDAY APRIL 23, 2014
TIME: 150 MINUTES

Name: ____

There are 10 questions, each worth 10 points. Partial credit is awarded where appropriate. Each solution must include enough detail to justify any conclusions you reach in answering the question.

1. A child is pulling a loaded toy-box on a level path with a force of 40 Newtons exerted at an angle of 60° above the horizontal.
   (a) [7 points] Find the horizontal and vertical components of the force.
   (b) [3 points] If the initial frictional resistance on the path to be overcome is 25 Newtons, will the child succeed in moving the toy-box?

2. Find an equation for the plane that passes through the point $(1, 1, 1)$ and contains the line

$$x = 2, \quad y = 2 + t, \quad z = 1 + t.$$

3. Calculate the definite integral

$$\int_0^3 x\,dx.$$

as a limit of Riemann sums, using equal-length sub-intervals of $[0,3]$. You may assume that

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

4. (a) [6 points] Find an antiderivative of the function

$$f(x) = \frac{(x+1)^2}{x},$$

and use it to evaluate the area below the graph of f between $x = 1$ and $x = 2$.

(b) [4 points] Let

$$h(t) = \int_0^{5t} \cos(x^2)\, dx.$$

Use the Fundamental Theorem of Calculus and the chain rule to find $h'(t)$. If $h(t)$ represents the position of a moving car at time t, calculate the velocity of the car at time $t = 0$.

5. An oil storage tank ruptures at time $t = 0$ and oil leaks out from the tank at a rate of

$$r(t) = 90\sqrt{1 + 3t}$$

liters per hour. How much oil leaks out in the first hour?

6. Use integration by parts to calculate the indefinite integral

$$\int \ln |x^2 + x - 2| \, dx.$$

7. If we define the function $F(s)$ by the improper integral

$$F(s) = \int_0^\infty f(x)e^{-sx}\,dx,$$

where $f(x) = e^{-x}$, calculate $F(s)$ for all $s > 0$.

8. (a) [7 points] Find the area bounded by the two curves $y = \sqrt{t}$ and $y = t^3$.
   (b) [3 points] Two cars both start on a journey at time $t = 0$. If the velocity of car A at time $t \geq 0$ is $\sqrt{t}$ and that of car B at the same time is t^3, how does the area in part (a) relate to the positions of the cars at time $t = 1$?

9. (a) [7 points] Use the the function $f(x) = 1/x$ and the integral test to show that the series

$$\sum_{n\geq 1} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots$$

is divergent. State the three conditions that f must satisfy for the integral test to be applicable

(b) [3 points] Ugh is a Stone Age stone man. His intention is to make a bridge across a ravine by stacking large bricks so that each brick is further out across the ravine than the brick below. The top brick has an overhang of $1/2$, the second brick overhangs by $1/4$, the third by $1/6$, and so on with the n-th brick (on the bottom) having an overhang of $1/(2n)$ over the cliff. His mathematician friend has told him that according to a center-of-gravity calculation the structure will not topple. Find the total overhang width out from the ravine edge. Is there a maximum width ravine that can, in theory, be crossed in this manner? Explain.

10. Consider the series

$$\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$$

for real numbers x.

(a) [6 points] Use the ratio test to determine the values x for which the series is absolutely convergent.

(b) [2 points] Does this mean that the series is convergent for these values of x? Explain.

(c) [2 points] Explain how one can use part (b) above to determine the limit of the sequence $\{a_n\}$ where

$$a_n = \frac{4^n}{n!}$$