MA 125-CT,  CALCULUS I
February 7, 2017

Name (Print last name first): ..............................................

[TEST I]

Show all your work! No partial credit will be given for the answer only!

PART I

Part I consists of questions. Clearly write your answer in the space provided after each question. Show all of your work!
All problems in Part I are 7 points each

Question 1

Use the definition of the derivative to show that the derivative of \( y = f(x) = 3x^2 + x \) is \( f'(x) = 6x + 1 \).

Question 2

Find the derivative of \( f(x) = x^3 \sin(x) \)

Answer: .....................
Question 3

Find the derivative of \( y = f(x) = \frac{x^2 + 1}{x + 1} \).

Answer: ..................

Question 4

Find the derivative of \( y = f(x) = \sqrt{x}(3x + x^2) \).

Answer: ..................

Question 5

Find the equation of the tangent line to the graph of \( y = f(x) = 2\tan(x) \) at the point \( a = 0 \).

Answer: ..................

Question 6

Show that the equation \( x \cos(x) + 1 = 0 \) has at least one solution on the interval \([0, \pi] \). Hint: use Intermediate Value Theorem.

Answer: ..................
Problem 1 (10 points)

Suppose that $S(t) = t^3 - 4t^2 + t$ is the position of a particle at time $t$ (in seconds) on a line.
Find:
(a) the velocity at time $t$,
(b) the displacement from $t = 0$ to $t = 2$,
(c) the displacement from $t = 2$ to $t = 3$.
Recall that the displacement could be positive or negative depending on the direction of movement.
Problem 2 (12 points)

Given the graph of the function \( y = f(x) \) above, find:

1. \( \lim_{x \to -1^-} f(x) = \)
2. \( \lim_{x \to -1^+} f(x) = \)
3. \( \lim_{x \to -1} f(x) = \)
4. \( \lim_{x \to 4^-} f(x) = \)
5. \( \lim_{x \to 4^+} f(x) = \)
6. \( \lim_{x \to 4} f(x) = \)
7. \( \lim_{x \to 5^-} f(x) = \)
8. \( \lim_{x \to 5^+} f(x) = \)
9. \( \lim_{x \to 5} f(x) = \)
10. \( \lim_{x \to \infty} f(x) = \)

11. State all intervals on which \( f(x) \) is continuous.
12. State all intervals where \( f(x) \) is differentiable.
Problem 3 (10 points)

Find all points on the graph of $f(x) = 2x^3 - 9x^2$ where the tangent line is parallel to the line $y = -12x$.

Problem 4 (10 points)

Define a function

$$f(x) = \begin{cases} x^2-x, & x < 1, \\ kx, & x \geq 1, \end{cases}$$

a) (8 points) Find a value of $k$ so that $f(x)$ is a continuous function for all real numbers $x$.

b) (2 points) For the value of $k$ you found, is $f(x)$ differentiable at $x = 1$? Briefly explain.
Problem 5 (10 points)

Evaluate the following limits. Like always, justify your answers.

1. \( \lim_{x \to \infty} \sqrt{x + 3} + \sqrt{x} \)

2. \( \lim_{x \to \infty} \sqrt{x + 3} - \sqrt{x} \)
Scratch paper