MA 126-E0 Summer 2006 Test 2

1. In each of the following, determine if the sequence converges or diverges. If it converges, find its limit.

(a)

$$\left\{ \frac{4^n}{5^{n+2}} \right\}$$

(b)

$$\left\{ \frac{(n+2)!}{n!} \right\}$$

2. In each of the following, determine whether the series converges or diverges. It is not necessary to determine the sum of the series in the case of convergence.

(a)

$$\sum_{n=1}^{\infty} (-1)^{n+1} n e^{-n}$$

(b)

$$\sum_{n=0}^{\infty} \frac{\pi^n}{3^{n+1}}$$

(c)

$$\sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$$

(d)

$$\sum_{n=1}^{\infty} \frac{(n+3)!}{n! 3^n}$$

3. Let $s = \sum_{k=1}^{\infty} 1/k^{3/2}$, and let s_n be the n^{th} partial sum of this series. Find n so that s_n approximates s to within $1/10$. It is not necessary to compute s_n for this n.

4. (a) Show that the series $\sum_{k=1}^{\infty} (-1)^{k+1}(1/k^3)$ converges.

(b) Determine n so that the n^{th} partial sum of this series approximates the sum of the series to within $1/100$.

5. Determine the radius of convergence of each of the following power series. If this radius is finite and positive, give the open interval of convergence of the series.

(a)

$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{n2^n} (x+5)^n$$

(b)

$$\sum_{n=0}^{\infty} \frac{3^n}{n+1} x^n$$

6. Expand $f(x) = e^{-x^2}$ in a Maclaurin series. What is the radius of convergence of this series?

7. Carry out the following steps to use a power series to approximate

$$\int_0^{1/3} \frac{x^2}{1+x^4} dx$$

to within 1/10000000.

(a) Begin with the geometric series $\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$ for $-1 < r < 1$. Let r$= -x^4$ to get a Maclaurin series for $\frac{1}{1+x^4}$, and multiply this series by x^2 to get the Maclaurin series for $\frac{x^2}{1+x^4}$.

(b) Write an infinite series for $\int_0^{1/3} \frac{x^2}{1+x^4} dx$ by integrating the Maclaurin series for $\frac{x^2}{1+x^4}$ term by term.

(c) Determine n so that the n^{th} partial sum of the series for $\int_0^{1/3} \frac{x^2}{1+x^4} dx$ approximates the value of this integral with an error of no more than 1/10000000.

Each problem is worth 16 points. In problem 1, each part is worth 8; in problem 2, each part is worth 4; in problems 4 and 5, each part is worth 8; in problem 7, (a) and (b) are worth 6 each, and (c) is worth 4 points.