MA 126 Summer 2006 Final Examination

Each problem is worth 9 points.

1. Evaluate $\int_1^4 \sqrt{x}\ln(x)dx$.

2. Evaluate $\int_1^2 x\sqrt{x-1}dx$.

3. Evaluate

$$\int_0^1 \frac{-5x^2+2x-1}{(x+2)(x^2+1)}dx.$$

4. Show that the following integral converges, or show that it diverges. In the case of convergence, evaluate the integral.

$$\int_2^\infty \frac{1}{(3x+1)^2}dx.$$

5. The graph of $y = 1 + x^2$ for $1 \leq x \leq 2$, is rotated about the $x-$ axis. Find the volume of the resulting solid.

6. For each of the following, determine whether the sequence converges or diverges. In the case of convergence, determine the limit.

(a)

$$\left\{ \frac{\sqrt{n}}{4+\sqrt{n}} \right\}$$

(b)

$$\left\{ \frac{(n+6)!}{n(n+2)!} \right\}$$

7. For each of the following, determine whether the series converges or diverges. In the case of convergence, it is not necessary to determine the sum of the series.

(a)

$$\sum_{k=0}^\infty \frac{4}{k^{7/3}}$$

(b)

$$\sum_{k=1}^\infty \left(\frac{\pi}{e}\right)^k$$

(c)

$$\sum_{k=0}^\infty (-1)^k \frac{k}{1+k^2}$$

(d)

$$\sum_{k=0}^\infty \cos(k)$$

8. For each of the following power series, determine the radius of convergence and the (open) interval of convergence.

(a)

$$\sum_{n=0}^{\infty} \frac{2^n}{n^2}\left(x-\frac{3}{2}\right)^n$$

(b)

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{\sqrt{n+1}}(x+6)^n$$

9. Let $f(x) = x^3 e^{-x^2}$.

(a) Use the Maclaurin series for $e^x$ to write the Maclaurin series for $f(x)$.

(b) Use the Maclaurin series for $x^3 e^{-x^2}$ to write an infinite series for $\int_0^{1/2} x^3 e^{-x^2}\,dx$.

(c) Determine $n$ so that the $n^{th}$ partial sum of the series for $\int_0^{1/2} x^3 e^{-x^2}\,dx$ approximates the integral with an error not exceeding 1/10000.

10. Find parametric equations for the line passing through the points $(1,-4,2)$ and $(6,-4,-4)$.

11. Find the equatio of the plane containing the points $(4,8,0)$, $(-3,2,7)$ and $(6,1,-9)$.

12. Find an expression for the angle between the line through $P$ and $Q$, and the line through $P$ and $R$, where

$$P = (-3,-1,6), Q = (1,-6,3) \text{ and } R = (8,1,1).$$

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