Syllabus
MA 587/687
Shannon Starr
3 January 2019

Contact Information
Class room: Education Building 132
Class times: 2pm – 3:15pm, Tuesdays and Thursdays
Instructor Starr’s Office: Campbell Hall 478a
Office Hours: Monday, Wednesday 2pm–3:15pm and by appointment
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Topics

The main topic for the course is finite state space Markov chains.

A finite state space Markov chain is a stochastic process, meaning a random sequence of variables \(X_1, X_2, \ldots\), such that each \(X_t\) takes values in a finite set \(\Omega = \{\omega_1, \ldots, \omega_n\}\). Roughly speaking, the probability for the chain to jump from \(X_t = \omega_i\) to \(X_{t+1} = \omega_j\) is given by a matrix \(P_{ij}\), for all times \(t\) and all \(i, j \in \{1, \ldots, n\}\). More precisely, the conditional probability is

\[
P(X_{t+1} = \omega_j \mid X_t = \omega_i, X_1 = \omega_{k(1)}, \ldots, X_{t-1} = \omega_{k(t-1)}) = P_{ij},\]

for any \(k(1), \ldots, k(t-1) \in \{1, \ldots, n\}\) (i.e., independently of the values of \(X_1, \ldots, X_{t-1}\)).

There is an overlap between the study of Markov chains and matrix analysis. So this is an opportunity for the two topics of probability theory and linear algebra to reinforce each other. Moreover, there are many applications of Markov chains, for example in theoretical computer science or “big data.” So there is another opportunity for probability theory and computing to reinforce each other. We will program using Matlab in this class. Every UAB student is entitled to free Matlab software due to the university’s site license. We will cover the necessary elements of programming in Matlab, including how to download in class.

A main result in Markov chains is that, under fairly general conditions, the distribution of the Markov chain at time \(t\) converges to a unique invariant measure as \(t \to \infty\), no matter what the starting point is. A more modern question is what is the rate of this convergence.

Another way to ask the question is as follows. Suppose one wishes to sample from a theoretical distribution (such as a uniformly random sample of a proper coloring of a graph \(G = (V, E)\) using \(q\) colors for some fixed \(q \in \mathbb{N} = \{1, 2, \ldots\}\)). Then one might start with a non-random element of the probability space, and then run a Markov chain to “mix up” this
non-random element with the allowed set of outcomes. How long does one need to let the Markov chain run on the computer to have a sample which is “close to” random (in the sense that it is close to being distributed according to the limiting measure, which is the stationary or invariant distribution)?

This is known as the mixing time for the Markov chain. That is the title of the textbook for the class

*Mixing Times for Markov Chains*, David Levin, Yuval Peres and Elizabeth Wilmer

We will also cover some other topics, in somewhat the same vein. One of the topics that we will cover will be the spectrum for a Markov transition matrix $P$. For example, we will learn how the spectral gap relates to the relaxation time, which is different than the mixing time.

We will also cover some concentration of measure results, and review the weak law of large numbers and the central limit theorem, to reinforce these two essential topics that students should have learned in a first course in probability theory. The three pillars of a first course in probability theory are: (i) conditional probability for example up through Bayes’s rule, (ii) the weak law of large numbers, and (iii) the central limit theorem. Conditional probability is the backbone of Markov chains, since it is how Markov chains are defined, via equation (1).

**Grades**

**Homework:** 30% There will be homework assignments about once every two weeks. You are encouraged to collaborate. But each student must write up their own work.

**Matlab midterm:** 10% One of the aims of the course will be to master enough skills in Matlab to simulate basic Markov chains, as well as calculating their distributions, and performing matrix computations. The Matlab midterm will be in the middle of February, in class.

**Theoretical midterm:** 20% There will be a second midterm on the more theoretical aspects of the class right after Spring break, in class.

**Final exam:** 40% There will be a cumulative final, that will cover both Matlab and theoretical aspects of the class.

All exams will be in-class exams. All exams are open-book and open-notes. You are expected to bring in your own laptops for the exams. For the Matlab exam, you will need to have already downloaded Matlab to your laptop, using the university’s freely available Matlab software (using the site license). You are not allowed to communicate with anybody during the exam by computer or any other device (except for me, the instructor, in case you have questions about the test while it is taking place).

There will also be a list of projects that can be done for up to 15% of extra credit. The projects will be due after the final exam, but before grades are due. Therefore, if your final exam score is not high enough for the grade you are trying for, you can turn in an extra credit project to try to raise the grade. But you are advised to begin the project early enough in the semester to get full credit on it. If the project is not completed satisfactorily it may only get partial credit.