MA 795-00 – Ergodic Theory and Information
Special Topics Course, Spring 2021, 1 – 6 credit hours
Syllabus

Instructor: Dr. Nándor Simányi

Class meets: TBA, twice a week, for 75 min.

Delivery method: Face-to-face, hybrid, or remote. Preferably face-to-face.

Office hours: TBA

We will have two 75 minute long meetings per week. In the majority of class meetings I will be covering new material, the rest will be devoted to discussing the homework (some hints for the outstanding exercises that are due later, plus thorough discussion of the already submitted homework), and these special class meetings also serve as Q&A sessions.

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Web site: http://people.cas.uab.edu/∼simanyi/MA-795/ (Populated later.)

Text. Detailed printed class notes will be handed out regularly

Homework will be assigned on a weekly or bi-weekly basis.

Prerequisite. Admission to the graduate program, or by my permission

Assessment Procedures. Student achievement will be assessed by the following measures: Regularly assigned homework problems, two midterm tests, and a comprehensive final exam. A numerical score is given on each of them.

Grading Policy. The percentage of the final numerical grade assigned to each item is as follows: final exam: 40%; two midterm tests: 15% each; homework 30%.

This is going to be an introductory course to ergodic theory and dynamical systems, with particular emphasis on the concepts of entropy and information.

Many systems in nature (in physics, chemistry, biology, etc.) evolve in time under the governance of ordinary diferential equations. Oftentimes the vector field defining the corresponding ODE is obtained as a skew-orthogonal complement (under a symplectic structure) of the gradient field of a Hamiltonian (an energy function).

Under such circumstances, there appears a natural invariant measure with respect to the solution flow: The geometric volume form generated by symplectic form itself. This mesure is called the Liouville-measure of the considered phase flow. The above phenomenon naturally gives rise to the following, fundamental concept of a measurable dynamical system: It is a probability space (M, μ) equipped with a measurable action of the additive group of reals $t \mapsto S^t$, $\{S^t : M \mapsto M\}$ preserving the measure μ on the phase space M. If, furthermore, a smooth manifold structure (often a Riemannian manifold structure) is given on the phase space M, so that each map S^t is smooth, then we are speakeing about a smooth dynamical system.

Our main goal in the first semester is two-fold:
(1) Introduction into the spectral properties of dynamical systems, the so-called Koopmanism, including the isomorphism, conjugacy, spectral invariants, systems with discrete spectra, etc.
(2) Get familiar with the concepts of partitions, sigma-algebras, information, the Shannon-entropy, and the Kolmogorov-Sinai entropy.

The course will be based on my regularly distributed handouts and on some selected chapters of two auxiliary textbookes:
1. Peter Walters: An Introduction to Ergodic Theory, Graduate Texts in Mathematics 79, Springer Verlag 1982.
2. Patricke Billingsley: Ergodic theory and information, John Wiley & Sons, 1965.

Syllabus

1. Measure-preserving transformations, recurrence, ergodicity, mixing, the Ergodic Theorem.

2. Spectral properties, isomorphism, conjugacy of dynamical systems.

3. Transformations with discrete spectrum. Group rotations.

4. Partitions, information, the Shannon entropy. The Kolmogorov-Sinai (metric) entropy of a transformation.

5. Topological dynamics: minimality, the non-wandering set, topological transitivity, topological conjugacy and discrete spectrum.

6. Invariant measures for continuous transformations. The Bogolyubov-Sinai Theorem on the existence of invariant measures. The extremal points of the convex, compact set of invariant measures. Expansive homeomorphisms.

7. Bowen's definition of topological entropy and its calculation.

8. Relationship between the topological and metric entropy. The Variational Principle.