EC 330 / ECO2007S
Combining simultaneous and sequential moves
D&S chapter 5

1. In many games we must combine extensive-form and strategic-form analyses. For example, agents with ongoing relationships often play sequences of simultaneous-move games over time.

2. CrossTalk and GlobalDialog game

   Two telecoms firms (CrossTalk and GlobalDialog) decide on their investment strategies.

   The game plays as follows:

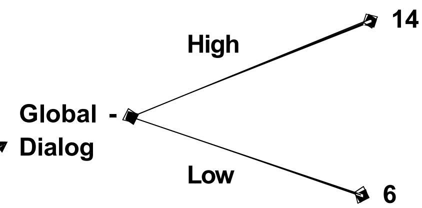
   - The firms (simultaneously) decide whether to invest
   - If one invests and the other does not, the investor can choose a high price or a low price.
   - The high price will attract 60 million customers, with an operating profit of $400 per customer.
   - The low price will attract 80 million customers, with an operating profit of $200 per customer.

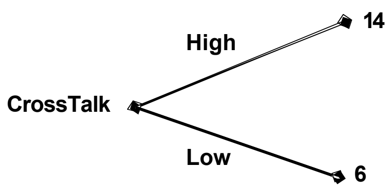
   - If both firms invest, they play a simultaneous pricing game (a Bertrand game).
   - If they both choose a high price or a low price, they split the market equally.
   - If they choose different prices, the firm with the low price gets the whole market.
   - It costs $10 billion to invest.

- So this is a sequential game consisting of two simultaneous-play rounds.
- Each step represents a subgame.

GlobalDialog

CrossTalk	Don't	Invest
Don't	0,0	0,?
Invest	?,0	?,?





GlobalDialog

CrossTalk	High	Low
High	2,2	-10,6
Low	6,-10	-2,-2

4. We can immediately calculate the payoffs to each firm in the cases where one invests and the other doesn't. Each firm would price high and make a profit of \$14 billion (\$400 × 60 million = \$24 billion; then subtract the \$10 billion cost of investment) since pricing low yields a profit of only \$6 billion. Since we assume the firms' utility functions are linear in money, we can eliminate these subgames and plug their outcomes into the appropriate cells on the stage-1 matrix.

5. But to do anything with the lower-right corner of the stage-1 matrix, we must analyze the stage-2 Bertrand game.

6. When both price high, each gets half the market. Thus each gets $400 × 30 million = $12 billion. Subtract the $10 billion investment cost. Profit per firm = $2 billion. In keeping with what we did above, call this payoff 2.

7. When both price low, each gets half of the larger market: $200 × 40 million = $8 billion. Subtract the $10 billion investment cost. Profit per firm = a loss of $2 billion. Call this -2.

8. When one firm prices high and the other prices low, the low-price firm gets all the customers. Its profit is then $200 × 80 million = $16 billion minus the $10 billion investment cost = $6 billion. Call this payoff 6. The other firm loses its investment. Call this payoff -10.

9. Now study the matrix of the stage-2 game.

		Global Dialog: High	Global Dialog: Low
Cross Talk	High	2,2	-10,6
	Low	6,-10	-2,-2

What familiar game is this? What is its NE?

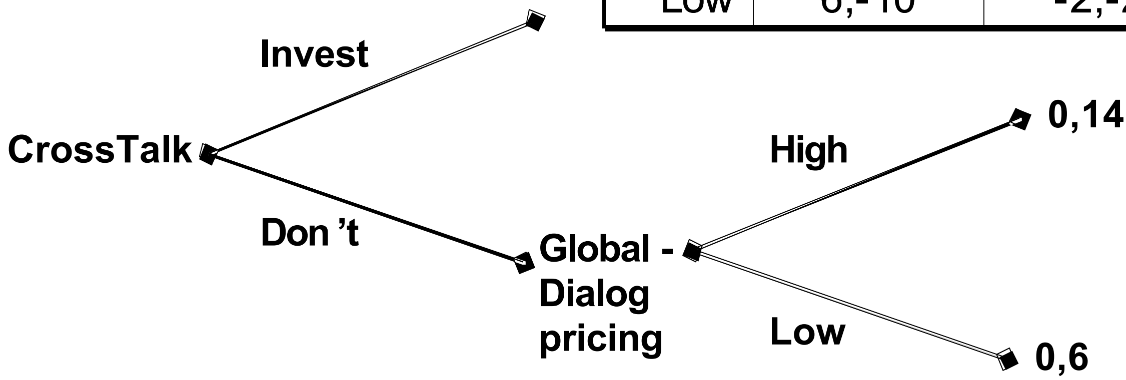
10. Now we can fill in the whole stage-1 matrix:

CrossTalk \ GlobalDialog	Don't	Invest
Don't	0,0	0,14
Invest	14,0	-2,-2

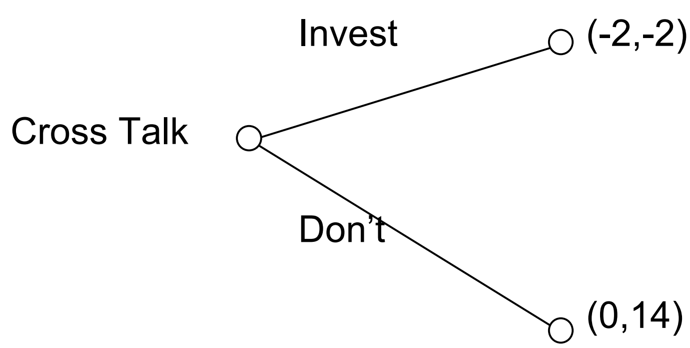
It's another familiar game. Which one?

11. Imagine that GlobalDialog has already invested and CrossTalk must now decide whether to enter. If it does, the firms play the pricing game as above. If it doesn't, GlobalDialog faces a simple pricing decision:

CrossTalk \ GlobalDialog	High	Low
High	2,2	-10,6
Low	6,-10	-2,-2



The solution:



Therefore CrossTalk won't invest and GlobalDialog gets the optimal payoff thanks to its first mover advantage. The structure of this game suggests that we might see a race to invest in this situation. Early investment is a commitment device here.

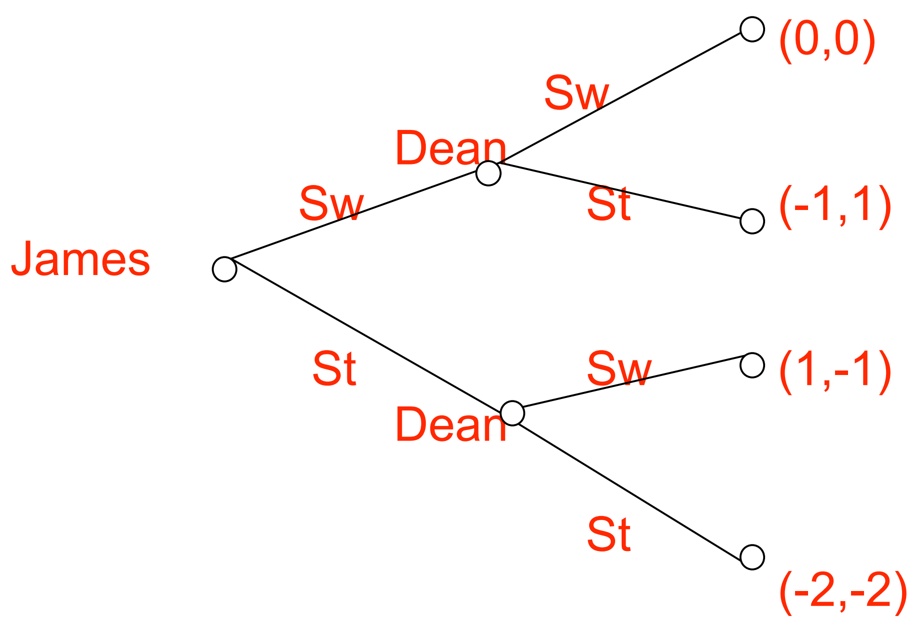
12. Changing games from simultaneous-move to sequential-move and vice-versa often changes outcomes (though it doesn't always do so, as we saw in the case of the PD). One can sometimes turn a sequential-move game into a simultaneous-move one by concealing actions; and one can sometimes effect the opposite transition by credible signaling.

13. In games with multiple NE in simultaneous-move versions, making the game sequential often allows the first mover to drive the game to the NE she prefers, as in Chicken:

Dean

James

	Swerve	Straight
Swerve	0,0	-1,1
Straight	1,-1	-2,-2

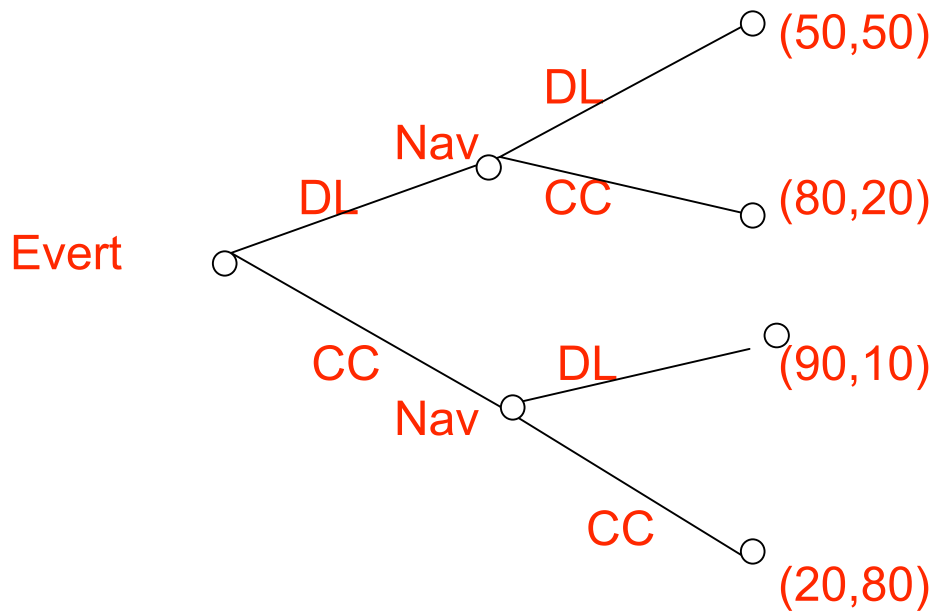


14. In other cases, such as our river crossing game and Dixit & Skeath's tennis example, making the games sequential produces second-mover advantages:

Navratilova

Evert

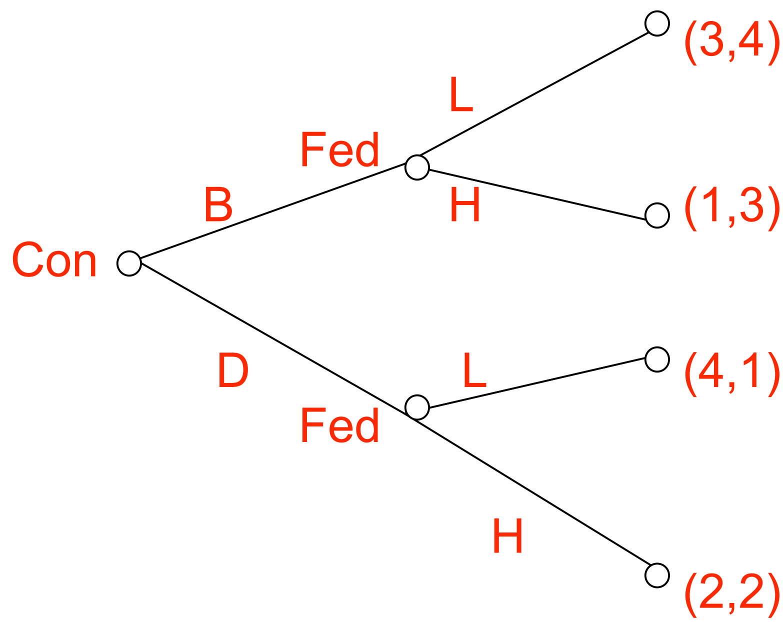
	DL	CC
DL	50	80
CC	90	20



15. In the Fed / Congress game from Chapter 4, changing the game to sequential and having Congress be Player I induces Congress to play a strategy that's dominated in the imperfect-information case, and turns the efficient outcome into the unique NE:

Fed

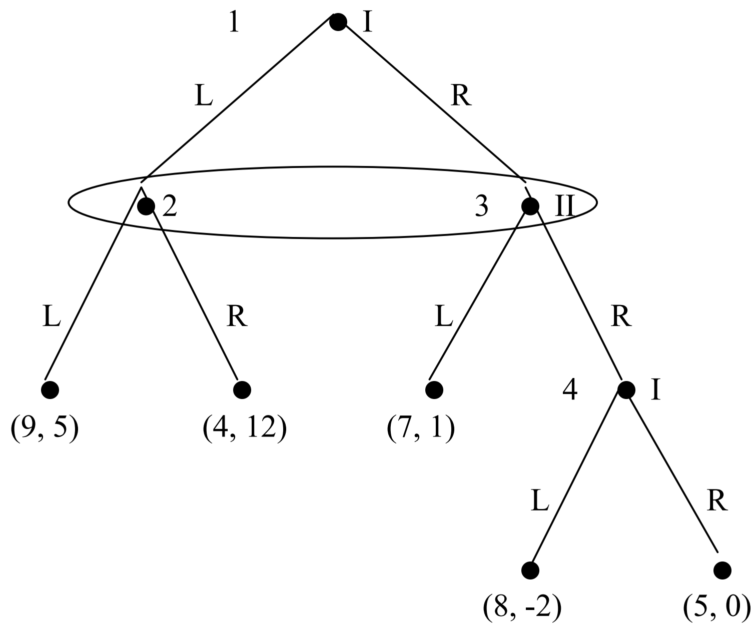
		Low i	High i
Congress	Balanced Budget	3, 4	1, 3
	Deficit	4, 1	2, 2



16. We can show simultaneous moves in extensive form – and in the process make an important philosophical point about the underlying nature of games in general.

17. We represent simultaneity in extensive form games by specifying *information sets*.

18. Consider the following extensive-form game (where action nodes are labeled with numerals for easy reference):



19. The oval drawn around nodes 2 and 3 indicates that they lie within a common information set. This means that at these nodes players cannot infer back up the path from whence they came; Player II does not know, in choosing her strategy, whether she is at 2 or 3. Put another way, II, when choosing, does not know what I has done at node 1. But this is just what defines two moves as simultaneous. We can thus see that the method of representing games in extensive form is entirely general. What players properly choose strategies over in *all* extensive-form games are, technically, information sets, rather than nodes themselves. If no node after the initial node is alone in an information set on its tree, so that the game has only one subgame (itself), then the whole game is one of simultaneous play. If at least one node shares its information set with another, while others are alone, the game involves both simultaneous and sequential play, and so is still a game

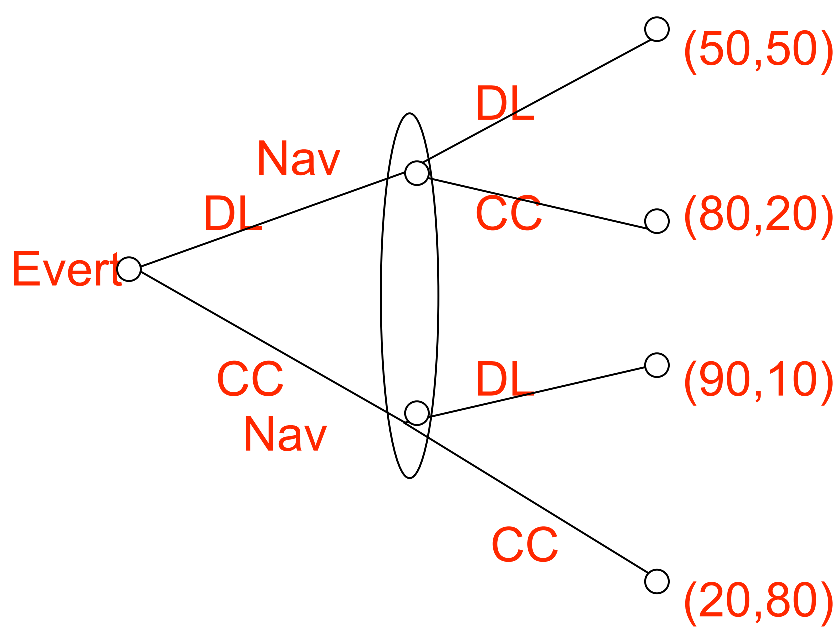
of imperfect information. Only if all information sets are singletons – inhabited by just one node – do we have a game of perfect information.

20. We now give a more formal definition of a subgame than we saw before:

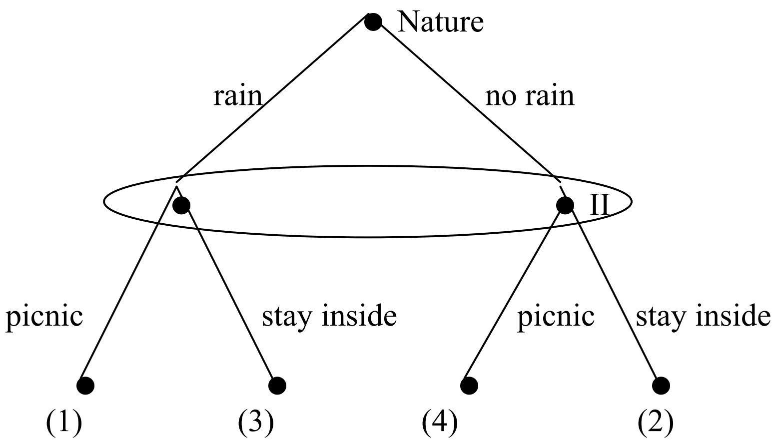
A subgame is a collection of nodes and branches that satisfy three properties:

i.) it starts at a single decision node
ii.) it contains every successor to this node
iii.) if it contains any part of an information set, then it contains all the nodes of the information set.

21. We can now display the tennis game as an extensive-form imperfect-information game:



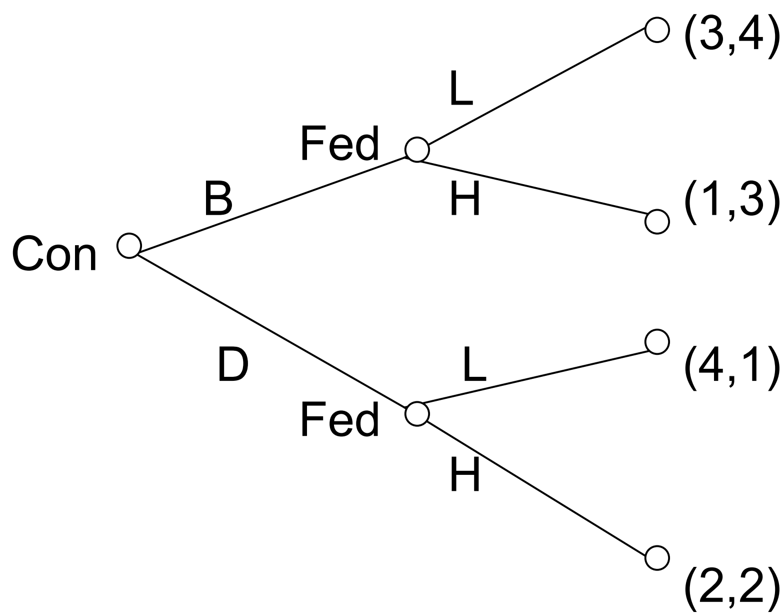
20. We also use this device to represent parametric uncertainty in games. Suppose you have to commit to a picnic before you know whether it's going to rain. What `nature' does here obviously affects your payoff. So we can introduce Nature into the game as an irrational Player 0 that chooses `rain' or `don't rain' with the probabilities indicated by the meteorological service. These outcomes both lie inside the information set from which you choose `picnic' or `no picnic':



How do we solve this? If the payoffs were cardinal, we'd multiply them by what the player believes to be the probabilities of rain and no rain to yield her *expected payoffs*. Suppose the player believes there's a 40% chance of rain. Then (if her payoffs above are cardinal), her expected payoffs at each terminal node, reading from left to right, are .4, 1.2, 2.4 and 1.2. The only NE here is in mixed strategies, which you'll learn to solve for next week. Suppose the expected payoffs looked like this instead: .8, .4, 1.6, 1.2. In that case you know the principle you need to solve for NE already. What's that principle?

In fact, though, you'd have to know the person wondering about a picnic very well to be confident that you could estimate cardinal payoffs in a situation like this. If you don't have cardinal payoffs you're confident about, but only ordinal ones, then you can't solve the above game for NE. Specifying its structure (as above) is as far as you can go. (Remember, though, that in this course the rule is that you can treat payoffs as cardinal if players are businesses and payoffs are in monetary prizes.)

21. Now we see how to represent a game of perfect information on a matrix. Recall the Fed / Congress game when Congress is first mover:



Congress has one move and two strategies. The Fed has two moves and four strategies: (L, H), (H, L), (L, L) and (H, H). So we can show all this on a 2 × 4 matrix, as follows:

Fed

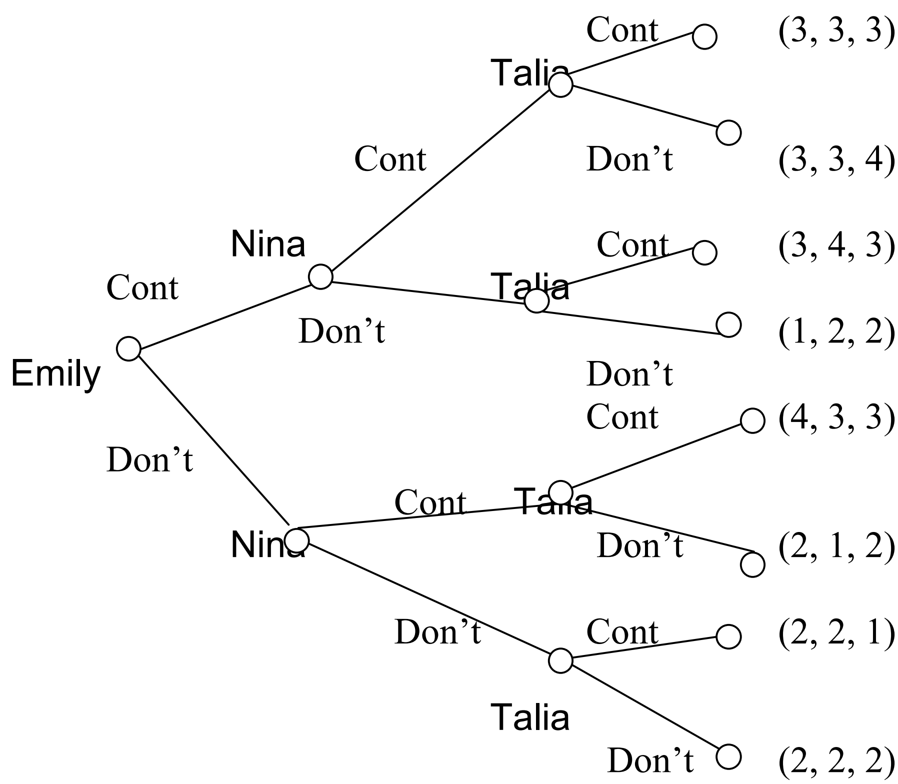
Cong

	L, H	H, L	L, L	H, H
Bal	*3,4*	1,3	3,4	1,3
Def	2,2	4,1	4,1	2,2

There are two pure-strategy NE: (BAL, (L, H)) and (Def, (H, H)).

22. Now, this is puzzling, because in extensive form this game had only one equilibrium: (BAL, (L, H)). How is it that it acquires a second NE in strategic form? The answer is that the equilibrium in the extensive form is `Nash plus': subgame perfect equilibrium (SPE) is a stronger solution concept than NE. NE requires that players do their best *given the other players' strategies.* SPE requires that players plan to do their best *for any possible vector of other players' strategies*. You should always use SPE where you can. Therefore, finite games of perfect information should always be represented in extensive form if they aren't purely simultaneous.

23. We now deploy our latest box of tricks on the common garden game. Recall its extensive form:



Now we show it as a simultaneous-move game:

Talia Contributes:

Nina

Emily

	C	D
C	3,3,3	*3,4,3*
D	*4,3,3*	2,2,1

Talia Doesn't Contribute:

Nina

Emily

	C	D
C	*3,3,4*	1,2,2
D	2,1,2	*2,2,2*

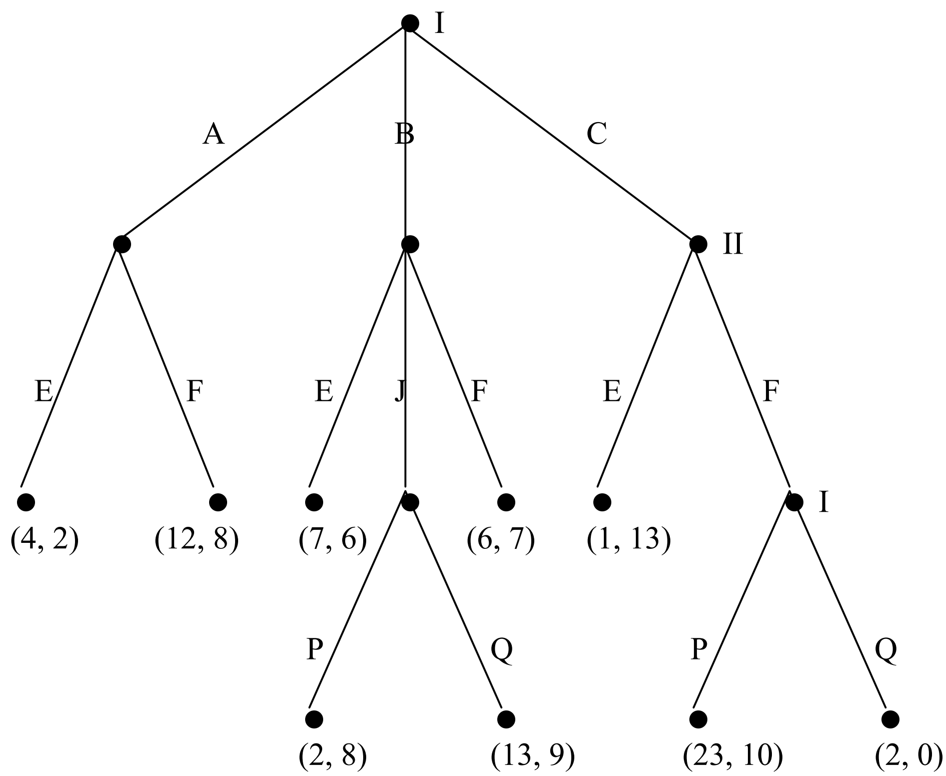
There are 4 NE: (D, C, C), (C, D, C), (C, C, D) and (D, D, D). It's easy to make intuitive sense of this. The first 3 NE correspond to the SPE of each possible extensive-form representation of the game – Emily first, Nina first, Talia first – in which the first mover plays D and the other two players play C. But now DDD is also a NE, because we allow the possibility of *no* first-mover advantage, in which case we have a Tragedy of the Commons.

24. Now we show the sequential-move version on a matrix. Emily has 2 strategies, so we'll make her Player III (the page player). Talia will now be Player I and Nina will remain Player II.

	EMILY							
	Cont				Dont			
	NINA				NINA			
TALIA	CC	CD	DC	DD	CC	CD	DC	DD
CCCC	3,3,3	3,3,3	3,4,3	3,4,3	3,3,4	1,2,2	3,3,4	1,2,2
CCCD	3,3,3	3,3,3	3,4,3	3,4,3	3,3,4	2,2,2	3,3,4	2,2,2
CCDC	3,3,3	3,3,3	3,4,3	3,4,3	2,1,2	1,2,2	2,1,2	1,2,2
CDCC	3,3,3	3,3,3	2,2,1	2,2,1	3,3,4	1,2,2	3,3,4	1,2,2
DCCC	4,3,3	4,3,3	3,4,3	3,4,3	3,3,4	1,2,2	3,3,4	1,2,2
CCDD	3,3,3	3,3,3	3,4,3	3,4,3	2,1,2	2,2,2	2,1,2	2,2,2
CDDC	3,3,3	3,3,3	2,2,1	2,2,1	2,1,2	1,2,2	2,1,2	1,2,2
DDCC	4,3,3	4,3,3	2,2,1	2,2,1	3,3,4	1,2,2	3,3,4	1,2,2
CDCD	3,3,3	3,3,3	2,2,1	2,2,1	3,3,4	2,2,2	3,3,4	2,2,2
DCDC	4,3,3	4,3,3	3,4,3	3,4,3	2,1,2	1,2,2	2,1,2	1,2,2
DCCD	4,3,3	4,3,3	3,4,3	3,4,3	3,3,4	2,2,2	3,3,4	2,2,2
CDDD	3,3,3	3,3,3	2,2,1	2,2,1	2,1,2	2,2,2	2,1,2	2,2,2
DCDD	4,3,3	4,3,3	3,4,3	3,4,3	2,1,2	2,2,2	2,1,2	2,2,2
DDCD	4,3,3	4,3,3	2,2,1	2,2,1	3,3,4	2,2,2	3,3,4	2,2,2
DDDC	4,3,3	4,3,3	2,2,1	2,2,1	2,1,2	1,2,2	2,1,2	1,2,2
DDDD	4,3,3	4,3,3	2,2,1	2,2,1	2,1,2	2,2,2	2,1,2	2,2,2

There are multiple NE here. But iterated elimination of dominated strategies, in the same order we considered players' moves using Zermelo's algorithm, isolates the SPE of the extensive-form game: (DCCD), (DC), (D).

25. Finally, we show how to represent asymmetric extensive-form games in strategic form. Consider:



By Zermelo's algorithm, the SPE here is (BQP, FJE).

Now here's the game's strategic form:

II

I	EEE	EEF	EJE	EJF	EFE	EFF	FEE	FEF	FJE	FJF	FFE	FFF
APP	4, 2	4, 2	4, 2	4, 2	4, 2	4, 2	12,8	12, 8	12,8	12,8	12, 8	12, 8
APQ	4, 2	4, 2	4, 2	4, 2	4, 2	4, 2	12,8	12, 8	12,8	12, 8	12, 8	12, 8
AQP	4, 2	4, 2	4, 2	4, 2	4, 2	4, 2	12,8	12, 8	12,8	12, 8	12, 8	12, 8
AQQ	4, 2	4, 2	4, 2	4, 2	4, 2	4, 2	12,8	12, 8	12,8	12, 8	12, 8	12, 8
BPP	7, 6	7, 6	2, 8	2, 8	6, 7	6, 7	7, 6	7, 6	2, 8	2, 8	6, 7	6, 7
BPQ	7, 6	7, 6	2, 8	2, 8	6, 7	6, 7	7, 6	7, 6	2, 8	2, 8	6, 7	6, 7
BQP	7, 6	7, 6	13,9	13,9	6, 7	6, 7	7, 6	7, 6	13,9	13, 9	6, 7	6, 7
BQQ	7, 6	7, 6	13,9	13,9	6, 7	6, 7	7, 6	7, 6	13,9	13, 9	6, 7	6, 7
CPP	1, 13	23,10	1,13	23,10	1,13	23,10	1,13	23,10	1,13	23,10	1,13	23,10
CPQ	1, 13	23,10	1,13	2,0	1,13	2,0	1,13	2,0	1,13	2,0	1,13	2,0
CQP	1, 13	23,10	1,13	23,10	1,13	23,10	1,13	23,10	1,13	23,10	1,13	23,10
CQQ	1, 13	23,10	1,13	2,0	1,13	2,0	1,13	2,0	1,13	2,0	1,13	2,0

Suppose that Player I's final two nodes were in a common information set. Then she'd have only two information sets over which to choose strategies instead of three, and the strategic form would change to:

I \ II	EEE	EEF	EJE	EJF	EFE	EFF	FEE	FEF	FJE	FJF	FFE	FFF
AP	4, 2	4, 2	4, 2	4, 2	4, 2	4, 2	12, 8	12, 8	12, 8	12, 8	12, 8	12, 8
AQ	4, 2	4, 2	4, 2	4, 2	4, 2	4, 2	12, 8	12, 8	12, 8	12, 8	12, 8	12, 8
BP	7, 6	7, 6	2, 8	2, 8	6, 7	6, 7	7, 6	7, 6	2,8	2, 8	6, 7	6, 7
BQ	7, 6	7, 6	13, 9	13, 9	6, 7	6, 7	7, 6	7, 6	13, 9	13, 9	6, 7	6, 7
CP	1, 13	23, 10	1, 13	23, 10	1, 13	23, 10	1, 13	23, 10	1, 13	23, 10	1, 13	23, 10
CQ	1, 13	2, 0	1, 13	2, 0	1, 13	2, 0	1, 13	2, 0	1, 13	2, 0	1, 13	2, 0

NE are (AP, FEE), (AP, FFE), (AQ, FEE), (AQ, FFE), (BQ, EJE), and (BQ, FJE). Player I's uncertainty gives rise to risk that has the effect of creating some new NE on the left-hand side of the tree.